1. $\underbrace{e^- + e^+}$ + photons

$\underbrace{e^- + e^+} \rightarrow$ materials

photons $\rightarrow \vec{E} + \vec{B}$

materials

classical

quantum

dielectric constant / refractive index

refractive index.

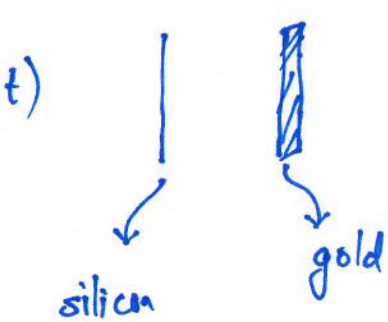
2. Classical ~~quantum~~ electrodynamics (Maxwell)

$$\vec{\nabla} \cdot \epsilon_0 \vec{E}(\vec{r},t) = \rho(\cancel{\vec{r}},t) - \cancel{\vec{\nabla} \cdot \vec{P}_{perm}}(\vec{r},t)$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0$$

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \times \frac{1}{\mu_0} \vec{H}(\vec{r},t) = \frac{\partial}{\partial t} \epsilon_0 \vec{E}(\vec{r},t) + \vec{j}(\cancel{\vec{r}},t) - \cancel{?}$$

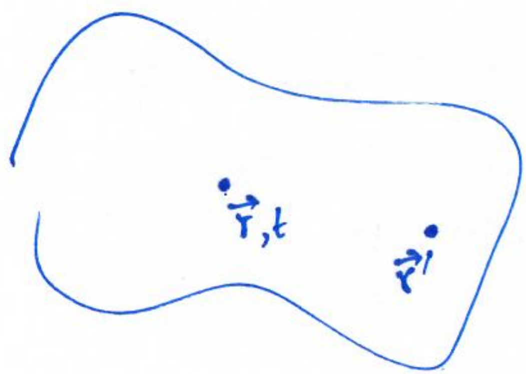


3 Induced polarization

$$\vec{P}(\vec{r},t) = \cancel{\int d^3r'} \int_{-\infty}^{t} dt' \overleftrightarrow{\chi}(\vec{r}, t-t') \cdot \epsilon_0 \vec{E}(\vec{r},t') \longleftarrow \underline{\underline{\text{model}}}$$

dyadic notation

$$P_i = \chi_{ij} E_j$$



Ginzberg
Electrodynamics.

4

$$\vec{P}(\vec{r},t) = \int_{-\infty}^{t} dt' \overleftrightarrow{\chi}(\vec{r}, t-t') \cdot \epsilon_0 \vec{E}(\vec{r},t)$$

$$= \int_{-\infty}^{+\infty} dt' \overleftrightarrow{\chi}(\vec{r}, t-t') \cdot \epsilon_0 \vec{E}(\vec{r},t).$$

$= 0 \quad t < t'$

$\chi = 0$

5 $$\vec{\nabla} \cdot \epsilon_0 \vec{E}(\vec{r},t) = \cancel{\chi} - \vec{\nabla} \cdot \int_{-\infty}^{+\infty} dt' \, \overleftrightarrow{\chi}(\vec{r},t-t') \cdot \epsilon_0 \vec{E}(\vec{r},t')$$

$$\vec{P}(\vec{r},t) = \int_{-\infty}^{+\infty} dt' \, \overleftrightarrow{\chi}(\vec{r},\underline{\underline{t-t'}}) \cdot \epsilon_0 \vec{E}(\vec{r},t')$$

$\downarrow$ Fourier transform

$$\vec{P}(\vec{r},\omega) = \overleftrightarrow{\chi}(\vec{r},\omega) \cdot \epsilon_0 \vec{E}(\vec{r},\omega)$$

Fourier transform of

$\vec{\chi}(\vec{r},t-t')$ ($t<t'$ is 0)

6 $$\vec{\nabla} \cdot \epsilon_0 \vec{E}(\vec{r},\omega) = - \vec{\nabla} \cdot \vec{P}(\vec{r},\omega)$$
$$= - \vec{\nabla} \cdot \overleftrightarrow{\chi}(\vec{r},\omega) \, \epsilon_0 \vec{E}(\vec{r},\omega)$$

$$\vec{\nabla} \cdot \left(\epsilon_0 + \epsilon_0 \chi \right) \vec{E} = 0$$

$$\vec{D} = \left[\overleftrightarrow{1} + \overleftrightarrow{\chi}(\vec{r}, \omega) \right] \cdot \epsilon_0 \vec{E}(\vec{r}, \omega)$$

$$\frac{\overleftrightarrow{\epsilon}(\vec{r}, \omega)}{\epsilon_0} - \overleftrightarrow{1} = \overleftrightarrow{\chi}(\vec{r}, \omega)$$

8 $$\overleftrightarrow{\chi}(t-t') = \begin{cases} \overleftrightarrow{f}(t-t') & t > t' \\ 0 & t < t' \end{cases} \leftarrow$$

$$= \theta(t-t')\,\overleftrightarrow{f}(t-t') \qquad \theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

9 $$\chi(t) = \theta(t)\, f(t)$$

(i) $\chi(t)$ has to real. $\quad f(t)^* = f(t)$

(ii) Since $f(t)$ $t<0$ arbitrary, choose $f(-t) = -f(t)$ (odd function)

(iii) causality $\quad \theta(t)$

Kramers - Kronig relation

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$$f(t)^* = f(t)$$

$$f(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, f(\omega)$$

$$f(t)^* = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{+i\omega t}\, f(\omega)^*$$

$\omega = -\omega'$

$d\omega = -d\omega'$

$$= \int_{+\infty}^{-\infty} -\frac{d\omega'}{2\pi}\, e^{-i\omega' t}\, f(-\omega')^*$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, f(-\omega)^*$$

$$f(-\omega)^* = f(\omega)$$

$$f(\omega)^* = f(-\omega)$$

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$$f(-t) = -f(t)$$

$$f(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, f(\omega)$$

$$f(-t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{+i\omega t}\, f(\omega) \qquad \omega \to -\omega$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, f(-\omega)$$

$$f(-\omega) = -f(\omega)$$

$$f(\omega)^{*} = f(-\omega)$$

(12) $\chi(t) = \theta(t)\, f(t)$

odd. $f(-\omega) = -f(\omega)$

$f(\omega) = f_r(\omega) + i\, f_i(\omega)$

real $f(\omega)^* = f(-\omega)$

real: $f_r(\omega) - i\, f_i(\omega) = f_r(-\omega) + i\, f_i(-\omega)$

$f_r(-\omega) + i\, f_i(-\omega) = -f_r(\omega) \mp i\, f_i(\omega)$

odd:

$$\left.\begin{array}{l} f_i(\omega) = -f_i(-\omega) \\ f_i(\omega) = -f(-\omega) \end{array}\right\}$$

$f_r(\omega) = f_r(-\omega)$

$f_r(\omega) = -f_r(-\omega)$

f_i is odd.

$\Rightarrow f_r = 0$

$f(\omega) = 0 + i\, f_i(\omega)$ $f_i(\omega)$ is odd.

13 $$\chi(t) = \theta(t)\, f(t)$$

$$(\chi\text{-real}) + \cancel{\chi}(f \text{ odd}) \Rightarrow f(\omega) = i\, f_i(\omega)$$

$$f(\omega) = i\, f_i(\omega)$$

14 $$\chi(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, \chi(t).$$

$$= \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, \theta(t)\, f(t)$$

$$= \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, e^{-i\omega' t}\, \theta(\omega') \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi}\, e^{-i\omega'' t}\, f(\omega'')$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi}\, \cancel{\int_{-\infty}^{+\infty}}\, f(\omega'')\, \theta(\omega') \int_{-\infty}^{+\infty} dt\, e^{it(\omega - \omega' - \omega'')}$$

15 $$\chi(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega''}{2\pi} f(\omega'')\, \theta(\omega')\, 2\pi\, \delta(\omega - \omega' - \omega'')$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, \theta(\omega')\, f(\omega - \omega')$$

$$f(\omega) = i\, f_i(\omega)$$

16 $$\theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \longrightarrow \theta(\omega) = \lim_{\delta \to 0} \frac{i}{\omega + i\delta}$$

$$\theta(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, \theta(t) \qquad \left.\frac{e^{i\omega t}}{i\omega}\right|_{t=0}^{t\to\infty}$$

$$= \int_0^{\infty} dt\, e^{i\omega t}\, e^{-\delta t}$$

$$= \lim_{\delta \to 0} \int_0^{\infty} dt\, e^{i(\omega + i\delta)t} = \lim_{\delta \to 0} \left.\frac{e^{i(\omega+i\delta)t}}{i(\omega + i\delta)}\right|_{t=0}^{t\to\infty}$$

$$= + \lim_{\delta \to 0} \frac{i}{\omega + i\delta}$$

17 $$\theta(\omega) = \frac{i}{\omega + i\delta} = \frac{i(\omega - i\delta)}{\omega^2 + \delta^2}$$

$$= \lim_{\delta \to 0} \frac{\delta}{\omega^2 + \delta^2} + i \frac{\omega}{\omega^2 + \delta^2}$$

$$= \pi\, \delta(\omega) + i \frac{\omega}{\omega^2 + \delta^2}$$

$$\frac{\delta}{\omega^2 + \delta^2} = 0 \;/\; \to \infty \;/\; \delta \to 0 \;/\; \omega \to 0 \quad \omega \to 0$$

$$\begin{cases} 0 & \omega \neq 0 \\ \to \infty & \omega = 0 \end{cases} = \pi\, \delta(\omega)$$

$$\int_{-\infty}^{+\infty} d\omega \frac{\delta}{\omega^2 + \delta^2} = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\delta^2 \sec^2\theta \, d\theta}{\delta^2 \sec^2\theta} = \pi$$

$\omega = \delta \tan\theta$

$d\omega = \delta \sec^2\theta \, d\theta$

(18) $$\chi(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, \theta(\omega')\, f(\omega - \omega')$$

$$\theta(\omega') = \pi\,\delta(\omega') + i\,\frac{\omega'}{\omega'^2 + \delta^2}$$

$$f(\omega - \omega') = i\, f_i(\omega - \omega')$$

(19) $$\chi(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \left[\pi\,\delta(\omega') + i\,\frac{\omega'}{\omega'^2 + \delta^2} \right] i\, f_i(\omega - \omega')$$

$$= \frac{i}{2} f_i(\omega) - \underbrace{\int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, \frac{\omega'}{\omega'^2 + \delta^2}\, f_i(\omega - \omega')}_{\mathrm{Re}\,\chi(\omega)}$$

$$\frac{1}{2} f_i(\omega) = \mathrm{Im}\,\chi(\omega)$$

$$\underline{\underline{\operatorname{Im} \chi(\omega)}} = \frac{1}{2} f_i(\omega)$$

$$\boxed{\operatorname{Re} \chi(\omega) = -\int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{(\omega'+\omega)}{\cancel{\omega'^2}\,(\omega'+\omega)^2 + \delta^2}\, 2\left[\operatorname{Im} \chi(\omega')\right]}$$

$\cancel{\omega - \omega'}\ \omega' - \omega = \omega''$

$d\omega' = d\omega''$

(20)
$$\chi(\omega) = \int \frac{d\omega'}{2\pi}\, \theta(\omega')\, f(\omega - \omega')$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, \theta(\omega'' + \omega)\, f(\omega'').$$

(21) <u>Example</u>: $\chi(\omega) = \dfrac{\omega_1}{\omega_0 - \omega} + i\pi\,\omega_1\, \delta(\omega - \omega_0)$

refractive

observation

$$\vec{P}(t) = \int_{-\infty}^{+\infty} dt'\,\chi(t-t')\,\vec{E}(t') + \int \theta \quad \vec{B}(t')$$