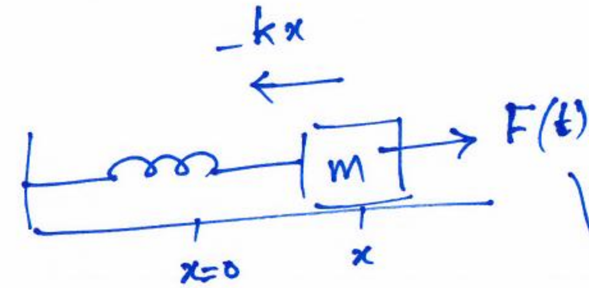
Date: 2024 Feb 17

1 Harmonic oscillator



fluctuations

$$ma = -kx + F$$

$$\left(m\frac{d^2}{dt^2} + k\right) x(t) = F(t)$$

$F(t) \rightarrow$ source.

(2) $$W[x;F] = \int_{t_1}^{t_2} dt \left[\frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - \frac{1}{2} k x^2 + F(t)\, x(t) \right]$$

$$\frac{\delta x(t)}{\delta x(t')} = \delta(t-t')$$

$$\frac{\delta W[x;F]}{\delta x(t)} = -m\frac{d^2}{dt^2} x(t) - k\,x(t) + F(t)$$

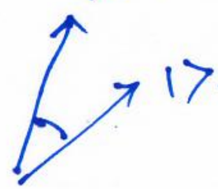
Stationary action principle $$\frac{\delta W}{\delta x(t)} = 0 \Rightarrow \left(m\frac{d^2}{dt^2} + k\right) x(t) = F(t)$$

$$\frac{\delta W}{\delta F(t)} = x(t)$$

3 Transition amplitude.

$$Z[F] = \langle \ | \ \rangle^{F}$$ → vacuum to vacuum transition amplitude.

$$\langle \ | \underbrace{x \ | \ \rangle}_{x|\rangle}$$



4 Schwinger's quantum action principle.

$$\delta Z = \frac{i}{\hbar} \langle \ | \ \delta W \ | \ \rangle$$

$$\cancel{\delta Z = \delta W \, Z}$$
$$\cancel{\ln Z = W}$$
$$\cancel{Z = e^{iW}}$$

$$\langle \ | \ \rangle$$

$$\int_{-\infty \ t_1}^{+\infty \ t_2} dt \, f(t)$$

(5) $$\frac{\delta W}{\delta x(t)} = -m\frac{d^2}{dt^2}x - kx + F$$

$$\left\langle \frac{\delta W}{\delta x(t)} \right\rangle = \left(-m\frac{d^2}{dt^2} - k\right)\langle x(t) \rangle^F + F(t)\langle 1 \rangle^F$$

$$0 = \frac{\hbar}{i}\frac{\delta Z}{\delta x(t)} = \left(-m\frac{d^2}{dt^2} - k\right)\langle x(t) \rangle^F + F(t)\langle 1 \rangle^F$$

Quantum action prin.

$$\left(+m\frac{d^2}{dt} + k\right)\boxed{\frac{\langle x(t) \rangle^F}{\langle 1 \rangle}} = F(t)$$

$$\langle x(t) \rangle$$

$$\frac{1}{Z}\frac{\hbar}{i}\frac{\delta Z}{\delta F(t)} = \frac{\langle x(t) \rangle}{\langle 1 \rangle} = \langle x(t) \rangle$$

(6) $$\left(m\frac{d^2}{dt^2}+k\right)\langle x(t)\rangle = F(t) \qquad \langle x(t)\rangle = \frac{\langle\, |x(t)|\,\rangle}{\langle 1\rangle}$$

$$\langle x(t)\rangle = \frac{1}{Z}\frac{\hbar}{i}\frac{\delta Z}{\delta F(t)}$$

(7) $$\left(m\frac{d^2}{dt^2}+k\right) G(t,t') = \delta(t-t') \rightarrow \delta_{ij}$$

$$G(t,t') = \left(m\frac{d^2}{dt^2}+k\right)^{-1}$$

$$\langle x(t)\rangle = \int_{-\infty}^{+\infty} dt'\, G(t,t')\, F(t')$$

(8) $$\frac{\delta}{\delta F(t')}\langle x(t)\rangle = G(t,t')$$

9 $$G(t,t') = \frac{\delta}{\delta F(t')} \frac{1}{Z[F]} \frac{\hbar}{i} \frac{\delta Z[F]}{\delta F(t)}$$

$$\frac{\hbar}{i} G(t,t') = \frac{1}{Z} \frac{\hbar}{i} \frac{\delta}{\delta F(t')} \frac{\hbar}{i} \frac{\delta}{\delta F(t)} Z - \left(\frac{1}{Z} \frac{\hbar}{i} \frac{\delta Z}{F(t')}\right)\left(\frac{1}{Z} \frac{\hbar}{i} \frac{\delta Z}{\delta F(t)}\right)$$

$\downarrow$ definition

$$\frac{\hbar}{i} G(t,t') = \langle x(t)\, x(t') \rangle^{F} - \langle x(t') \rangle^{F} \langle x(t) \rangle^{F}$$

correlations

10 Quantum fluctuations

$$\langle x(t) \rangle^{F\to 0} = 0$$

$$\boxed{\langle x(t)\, x(t') \rangle = \frac{\hbar}{i} G(t,t')}$$

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$$\left(m\frac{d^2}{dt^2} + k\right) x(t) = F(t)$$

$$x(t) = \int_{-\infty}^{+\infty} dt'\, G(t,t')\, F(t')$$

$$\langle x(t)\, x(t')\rangle = \int d\tau \int d\tau'\; G(t,\tau)\, G(t',\tau') \underbrace{\langle F(\tau)\, F(\tau')\rangle}_{\delta(\tau-\tau')}$$

$$= \int_{-\infty}^{+\infty} d\tau\; G(t,\tau)\, G(t',\tau)$$

12 $$\langle x(t) \rangle = \int_{-\infty}^{+\infty} dt' \, G(t,t') \, F(t') \qquad F$$

$$\frac{\hbar}{i} \frac{\delta Z}{\delta F(t)} = Z[F] \int_{-\infty}^{+\infty} dt' \, G(t,t') \, F(t')$$

$$Z[F] = Z[0] \, e^{\frac{i}{\hbar} \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' \, F(t) \, G(t,t') \, F(t')} \qquad \tfrac{1}{2}$$

13 Quantum electrodynamics.

$$\langle \vec{E}(\vec{x},t) \rangle = 0$$

$$\langle \vec{E}(\vec{x},t) \, \vec{E}(\vec{x}',t') \rangle = \frac{\hbar}{i} \overleftrightarrow{\Gamma}(\vec{x}, \vec{x}', t-t')$$

$\xrightarrow{\text{Quantum}}$ Green's dyadic

1 diphanon
2 Blackhole.

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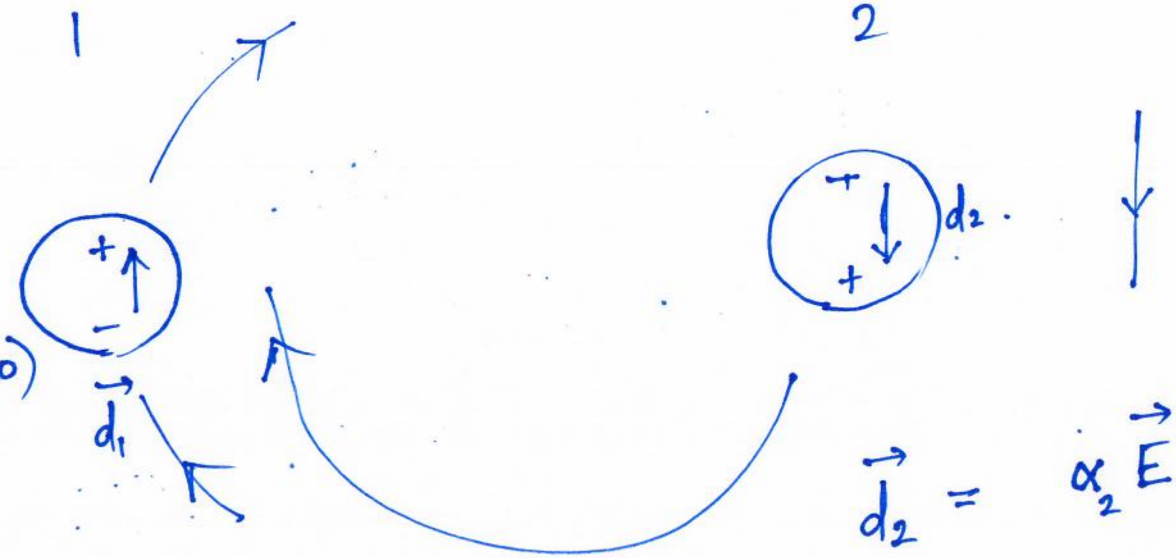
diphanam.

Jain.

John D. Barrow. (~2020)
Entropy of blackhole.
Phys. Lett.

E_2



$\vec{E_1} = \frac{d_1}{r^3}$

$\vec{d_2} = \alpha_2 \vec{E_1}$

$E_2 = \frac{d_2}{r^3}$

$d_1 \cancel{d_1'}$ $\quad d_1' = \alpha_1 E_2$

$= \alpha_1 \frac{\alpha_2 E_1}{r^3} = \frac{\alpha_1 \alpha_2 d_1}{r^6}$

$\langle d_1\, d_1' \rangle$?

$= \langle \alpha_1\, \alpha_2 \rangle$

$= kT$

Energy $= - d_1' \cdot E_2 = - d_1' \cdot \frac{d_2}{r^3} = - d_1'$