2024 Mar 2

①

① $$\mathcal{L} = -\frac{1}{2}\vec{E}^{*}\cdot\overleftrightarrow{\epsilon}\cdot\vec{E} + \vec{E}^{*}\cdot\overleftrightarrow{\epsilon}\cdot\left[-\vec{\nabla}\phi + i\omega\vec{A}\right] + \frac{1}{2}\vec{H}^{*}\cdot\overleftrightarrow{\mu}\cdot\vec{H} - \vec{H}^{*}\cdot\vec{\nabla}\times\vec{A} + \vec{P}^{*}\cdot\left[-\vec{\nabla}\phi + i\omega\vec{A}\right] - \left(\cancel{\rho}\,\phi - \cancel{\vec{J}}\cdot\vec{A}\right)$$

② $\left.\begin{array}{l}\vec{E}(\vec{r},\omega),\ \vec{H}(\vec{r},\omega)\\ \phi(\vec{r},\omega),\ \vec{A}(\vec{r},\omega)\end{array}\right\}$ dynamical fields

$\overleftrightarrow{\epsilon}(\vec{r},\omega)$, $\overleftrightarrow{\mu}(\vec{r},\omega)$ – background potentials

$\vec{P}(\vec{r},\omega)$, $\vec{M}\cancel{(\vec{r},\omega)}$ – source function

→ dual symmetry.
(refer: Milton et al. ~2008)

3 $$W = \int d^3r \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \mathcal{L}$$ $\cancel{\int dE}$

vacuum to vacuum transition amplitude.

$$Z[\vec{P}] = \langle 0_+ | 0_- \rangle^{\vec{P}}$$

$$H(\vec{r}, \vec{P}) = \frac{P^2}{2m}$$

$$\times \quad Z \leftrightarrow e^{\frac{i}{\hbar} W} \leftarrow$$

$$\leftrightarrow \frac{1}{2} m v^2$$

$$\delta Z = \frac{i}{\hbar} \langle 0_+ | \delta W | 0_- \rangle^{\vec{P}}$$

Schwinger's quantum action principle.

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4 $\delta \vec{E}$: $\quad \vec{E} = -\vec{\nabla}\phi + i\omega \vec{A} \Rightarrow \vec{\nabla}\times\vec{E} = i\omega\vec{\nabla}\times$

$\delta \vec{H}$: $\quad \vec{B} = \vec{\nabla}\times\vec{A} \Rightarrow \vec{\nabla}\cdot\vec{B} = 0$

$\vec{D} = \overleftrightarrow{\epsilon}\cdot\vec{E}$

$\delta \phi$: $\quad \vec{\nabla}\cdot(\vec{D}+\vec{P}) = \cancel{\rho}\ 0$

$\vec{B} = \overleftrightarrow{\mu}\cdot\vec{H}$

$\delta \vec{A}$: $\quad \vec{\nabla}\times\vec{H} = -i\omega(\vec{D}+\vec{P}) + \cancel{\vec{j}}$

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$$\vec{\nabla}\times\vec{E} = i\omega\vec{B} \Rightarrow \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = -i\omega(\vec{D}+\vec{P}) \Rightarrow \vec{\nabla}\cdot(\vec{D}+\vec{P}) = 0$$

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$$\boxed{\frac{\delta W}{\delta \vec{P}(\vec{r},\omega)^*} = \vec{E}(\vec{r},\omega)\,\frac{1}{2\pi}}$$

$$\delta W = \int d^3r \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\; \delta \vec{P}^* \cdot \vec{E}$$

$$\delta Z = \frac{i}{\hbar} \int d^3r \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\; \delta \vec{P}^* \cdot \langle 0_+ | \vec{E} | 0_- \rangle^{\vec{P}}.$$

$$\frac{\hbar}{i}\frac{\delta}{\delta P} \ln Z = \frac{1}{Z}\,\frac{\hbar}{i}\,\frac{\delta Z}{\delta \vec{P}^*(\vec{r},\omega)} = \frac{\langle 0_+ | \vec{E} | 0_- \rangle}{\langle 0_+ | 0_- \rangle} \equiv \vec{E}[\vec{r},\omega;\vec{P}]$$

$$E = \vec{\Gamma}\vec{P}$$

$$\downarrow$$

$$Z \Leftrightarrow e^{\frac{i}{\hbar}\iint \vec{P}^* \cdot \vec{E}\,\frac{1}{2}}$$

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$$\frac{\delta W}{\delta \overleftrightarrow{\epsilon}_{ij}(\vec{r},\omega)} = +\frac{1}{2}\, \vec{E}^{*}_{i}(\vec{r},\omega) \quad \vec{E}_{j}(\vec{r},\omega)\ \frac{1}{2\pi}$$

$$\frac{\delta Z}{\delta \overleftrightarrow{\epsilon}(\vec{r},\omega)} = \frac{i}{\hbar}\ \frac{1}{2} \left\langle 0_+ \middle| \vec{E}^{*}(\vec{r},\omega)\, \vec{E}(\vec{r},\omega) \middle| 0_- \right\rangle^{\vec{p}}_{\frac{1}{2\pi}}$$

$$\frac{\hbar}{i}\ \frac{\delta Z}{\delta \overleftrightarrow{\epsilon}(\vec{r},\omega)} = \frac{1}{2} \left\langle 0_+ \middle| \vec{E}^{*}(\vec{r},\omega)\, \vec{E}(\vec{r},\omega) \middle| 0_- \right\rangle^{\vec{p}}_{\frac{1}{2\pi}}$$

8 Recall

$$\frac{\hbar}{i}\frac{\delta}{\delta \vec{P}(\vec{r},\omega)}\frac{1}{Z}\frac{\hbar}{i}\frac{\delta}{\delta \vec{P}(\vec{r}',\omega)} Z = \langle 0_+ | \vec{E}^*(\vec{r}) \vec{E}(\vec{r}') | 0_- \rangle \frac{1}{2\pi}$$

$$\langle \vec{E} \rangle = 0$$

9 If $\langle \vec{E} \rangle = 0$

$$\frac{\hbar}{i}\frac{\delta Z}{\delta \overleftrightarrow{\epsilon}(\vec{r},\omega)} = \int d^3r\, \delta^{(3)}(\vec{r}-\vec{r}')\, \frac{\hbar}{i}\frac{\delta}{\delta \vec{P}(\vec{r},\omega)} \wedge \frac{\hbar}{i}\frac{\delta}{\delta \vec{P}(\vec{r}',\omega)} Z.$$

$$\langle \vec{d}\,\vec{d} \rangle = \frac{\hbar}{i}\alpha$$

$$\vec{P}(\vec{r},\omega)\, \vec{P}(\vec{r}',\omega)\Big|_{eff} = \frac{\hbar}{i}\delta \overleftrightarrow{\epsilon}(\vec{r},\omega)\, \delta^{(3)}(\vec{r}-\vec{r}')$$

$\overleftrightarrow{\mu} = \overleftrightarrow{I}\mu_0$ ⑦

$\langle \quad \rangle$

⑩ $$\vec{\nabla} \times \epsilon_0 \vec{E} = i\omega\mu_0\epsilon_0 \vec{H}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \epsilon_0 \vec{E}) = \frac{i\omega}{c^2} \; -i\omega(\vec{D} + \vec{P})$$

$$= \frac{\omega^2}{c^2}\left[\frac{\overleftrightarrow{\epsilon}}{\epsilon_0} \cdot \epsilon_0 \vec{E} + \vec{P}\right]$$

$$= \frac{\omega^2}{c^2}\left[\underbrace{\left(\frac{\overleftrightarrow{\epsilon}}{\epsilon_0} - \overleftrightarrow{1}\right)}_{\overleftrightarrow{\chi}} \cdot \epsilon_0 \vec{E} + \overleftrightarrow{1} \cdot \epsilon_0 \vec{E} + \vec{P}\right]$$

$$\left[\frac{c^2}{\omega^2} \vec{\nabla} \times (\vec{\nabla} \times \overleftrightarrow{1}) - \overleftrightarrow{1} - \overleftrightarrow{\chi}\right] \cdot \epsilon_0 \vec{E}(\vec{r}, \omega) = \vec{P}(\vec{r}, \omega)$$

$$\left(\frac{1}{Z} \frac{\hbar}{i} \frac{\delta Z}{\delta \vec{P}}\right)$$

11 Green's dyadic

$$\left[\frac{c^2}{\omega^2}\vec{\nabla}\times(\vec{\nabla}\times\overleftrightarrow{I}) - \overleftrightarrow{I} - \overleftrightarrow{\chi}\right]\cdot\overleftrightarrow{\Gamma}(\vec{r},\vec{r}\,';\omega) = \overleftrightarrow{I}\,\delta^{(3)}(\vec{r}-\vec{r}\,')$$

$$\epsilon_0\vec{E}(\vec{r},\omega) = \int d^3r'\,\overleftrightarrow{\Gamma}(\vec{r},\vec{r}\,';\omega)\cdot\vec{P}(\vec{r}\,',\omega)$$

$$\frac{1}{Z}\frac{\hbar}{i}\frac{\delta Z}{\delta\vec{P}(\vec{r},\omega)} = \int d^3r'\,\overleftrightarrow{\Gamma}(\vec{r},\vec{r}\,',\omega)\cdot\vec{P}(\vec{r}\,',\omega)$$

$$\frac{\hbar}{i}\frac{\delta}{\delta\vec{P}(\vec{r}\,',\omega)}\frac{1}{Z}\frac{\hbar}{i}\frac{\delta Z}{\delta\vec{P}(\vec{r},\omega)} = \frac{\hbar}{i}\overleftrightarrow{\Gamma}(\vec{r},\vec{r}\,',\omega)$$

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$$\frac{\delta}{\delta \overleftrightarrow{\epsilon}} \longleftrightarrow \int \delta \frac{\delta}{\delta \vec{P}} \quad \frac{\delta}{\delta \vec{P}} \quad \frac{1}{2}$$

$$\int d^3r\, \delta^{(3)}(\vec{r}-\vec{r}\,') \frac{\hbar}{i} \frac{\delta}{\delta \vec{P}(\vec{r}\,',\omega)} \frac{1}{Z} \frac{\hbar}{i} \frac{\delta Z}{\delta \vec{P}(\vec{r},\omega)} = \frac{\hbar}{i} \cancel{\int d^3r}\, \overleftrightarrow{\Gamma}(\vec{r},\vec{r},\omega)$$

$$\frac{1}{Z} \frac{\hbar}{i} \frac{\delta Z}{\delta \overleftrightarrow{\epsilon}(\vec{r},\omega)} = \frac{1}{2} \frac{\hbar}{i} \overleftrightarrow{\Gamma}(\vec{r},\vec{r},\omega)$$

$$\delta W \leftrightarrow \frac{\hbar}{i} \frac{\delta Z}{Z} = \frac{1}{2} \frac{\hbar}{i} \int d^3r \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \delta \overleftrightarrow{\epsilon} \cdot \overleftrightarrow{\Gamma}(\vec{r},\vec{r},\omega)$$

13 $\left[\frac{c^2}{\omega^2}\vec{\nabla}\times\vec{\nabla} - 1 - \underset{\cdot}{\chi}\right]\cdot\Gamma = 1$ $\quad [\ \ ] = \Gamma^{-1}$ 10

$\chi = \frac{\epsilon}{\epsilon_0} - 1$

$\delta\chi = \delta\epsilon$

$$-\delta\overleftrightarrow{\chi}\cdot\overleftrightarrow{\Gamma} + [\qquad]\cdot\delta\overleftrightarrow{\Gamma} = 0$$

$$\mathrm{tr}\ \delta\overleftrightarrow{\chi}\cdot\overleftrightarrow{\Gamma} = \overleftrightarrow{\Gamma}^{-1}\cdot\delta\overleftrightarrow{\Gamma} \qquad \frac{1}{\Gamma}\delta\Gamma$$

$$= \mathrm{tr}\,\delta\ \ln\overleftrightarrow{\Gamma}$$

Energy × time

14 $\delta W \leftrightarrow \frac{\hbar}{i}\frac{\delta Z}{Z} = \frac{1}{2}\frac{\hbar}{i}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\ \mathrm{tr}\ \ln\overleftrightarrow{\Gamma}$

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$$\frac{1}{2}\vec{E}_i \cdot \overleftrightarrow{\epsilon}_{ij} \cdot \vec{E}_j$$

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$$\frac{1}{Z}\frac{\hbar}{i}\frac{\delta Z}{\delta \overleftrightarrow{\epsilon}_{ij}} = \frac{1}{2}\frac{\hbar}{i}\ \overleftrightarrow{\Gamma}_{ij}$$

$$\frac{\hbar}{i}\ \frac{\delta Z}{Z} = \frac{1}{2}\frac{\hbar}{i}\int d^3r \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\ \mathrm{tr}\left[\delta \overleftrightarrow{\epsilon} \cdot \overleftrightarrow{\Gamma}\right]$$

$\Gamma(\vec{r},\vec{r})$

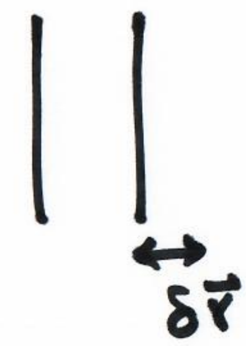
index

$$= \frac{1}{2}\frac{\hbar}{i}\int d^3r \int \frac{d\omega}{2\pi}\ \mathrm{tr}\ \overleftrightarrow{\Gamma}^{-1}\,\delta\overleftrightarrow{\Gamma}$$

$$= \frac{1}{2}\frac{\hbar}{i}\int \frac{d\omega}{2\pi}\ \mathrm{tr}\ \delta \ln \overleftrightarrow{\Gamma}$$

index + spatial!

16 $$\delta \overleftrightarrow{\epsilon} = \overleftrightarrow{\epsilon}(\vec{r}+\delta\vec{r},\omega) - \overleftrightarrow{\epsilon}(\vec{r},\omega)$$

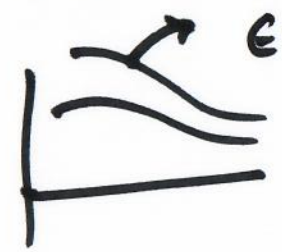


$$= \underline{\underline{\delta\vec{r}\cdot\vec{\nabla}\,\overleftrightarrow{\epsilon}(\vec{r},\omega)}}$$

$$\delta W = \int\int \delta\vec{r}\cdot\vec{\nabla}\,\overleftrightarrow{\Gamma}$$

$$\delta U = -\delta\vec{r}\cdot\vec{F}$$

17 $$\delta \overleftrightarrow{\epsilon} = \overleftrightarrow{\epsilon}(\vec{r},\omega+\delta\omega) - \overleftrightarrow{\epsilon}(\vec{r},\omega)$$



$$\delta E[\overleftrightarrow{\epsilon}] = E[\epsilon+\delta\epsilon] - E[\epsilon]$$

Casimir energy