Date: 2025 May 23

1

$$\vec{\nabla} \cdot \vec{D}(\vec{r},t) = \cancel{\rho(\vec{r},t)}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0$$

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial}{\partial t}\vec{B}(\vec{r},t)$$

$$\vec{\nabla} \times \vec{H}(\vec{r},t) = \frac{\partial}{\partial t}\vec{D}(\vec{r},t) + \cancel{\vec{J}(\vec{r},t)}$$

causality

$$\vec{P}(\vec{r},t) = \int_{-\infty}^{t} dt' \, \chi(\vec{r},t-t') \, \epsilon_0 \vec{E}(\vec{r},t')$$

$\boxed{\text{susceptibility}}$

$$\vec{D}(\vec{r},t) = \epsilon_0 \vec{E}(\vec{r},t) + \vec{P}(\vec{r},t)$$

$$\vec{B}(\vec{r},t) = \mu_0 \vec{H}(\vec{r},t)$$

$$P = \int \chi E$$

2

$$\frac{\partial}{\partial t} \to -i\omega$$

$$\vec{\nabla} \cdot \vec{D}(\vec{r},\omega) = \cancel{\rho(\vec{r},\omega)}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r},\omega) = 0$$

$$\vec{\nabla} \times \vec{E}(\vec{r},\omega) = i\omega \vec{B}(\vec{r},\omega)$$

$$\vec{\nabla} \times \vec{H}(\vec{r},\omega) = -i\omega \vec{D}(\vec{r},\omega) + \cancel{\vec{j}(\vec{r},\omega)}$$

where $\vec{D}(\vec{r},\omega) = \epsilon(\vec{r},\omega)\,\vec{E}(\vec{r},\omega)$

+ ϵ satisfy Kramers-Kronig relation

$$\vec{B}(\vec{r},\omega) = \mu_0 \vec{H}(\vec{r},\omega)$$

3 Quantum fluctuations

$\vec{P} = \int \chi \vec{E}$ 3

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$\rho = 0$ & $\vec{j} = 0$

mathematical tool for

$\langle x \rangle$ = average

$\vec{D} \to \vec{D} + \vec{P}_f$

source of quantum fluctuations

$\langle x\, x \rangle$ = variance

$$\left\langle \vec{E}(\vec{r},\omega)\, \vec{E}(\vec{r}',\omega) \right\rangle = \frac{\hbar}{i} \overleftrightarrow{\Gamma}(\vec{r},\vec{r}',\omega) \frac{1}{\epsilon_0}$$

correlation

Green's dyadic.

consistency

$$\left\langle \vec{P}_f(\vec{r},\omega)\, \vec{P}_f(\vec{r}',\omega) \right\rangle = \frac{\hbar}{i} \chi(\vec{r},\omega) \frac{1}{\epsilon_0} \delta^{(3)}(\vec{r}-\vec{r}')$$

4 $\mathcal{L} = \frac{1}{2}\phi\,\partial^2\phi + \phi(x)K(x)$ $\qquad W = \int dt\,\mathcal{L}.$

field. source

$$\frac{\delta \mathcal{L}}{\delta K(x)} = \phi(x)$$

$$W = \int dt\,\mathcal{L}$$

$$\delta W = \int dt\,[\delta\vec{r}\cdot\vec{P} - H\,\delta t]$$

translation generator of translation

$$\frac{\delta W}{\delta \vec{r}} = \dot{\vec{P}}$$

5

5

$$\vec{B} = \mu_0 \vec{H}$$
$$\vec{D} = \epsilon_0 \frac{\epsilon}{\epsilon_0} \vec{E}$$

$$\vec{\nabla} \times \vec{E} = i\omega \vec{B}$$
$$\vec{\nabla} \times \vec{H} = -i\omega (\vec{D} + \vec{P})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \mu_0 \frac{i\omega}{\cancel{\mu_0}} (-i\omega) (\vec{D} + \vec{P})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \underbrace{\omega^2 \mu_0 \epsilon_0}_{\omega^2/c^2} \left[\frac{\epsilon}{\epsilon_0} \vec{E} + \vec{P}\right]$$

$$= \frac{\omega^2}{c^2} \left[(\chi + 1) \vec{E} + \vec{P}\right] \quad \underbrace{\frac{\epsilon}{\epsilon_0} - 1}_{\chi} + 1$$

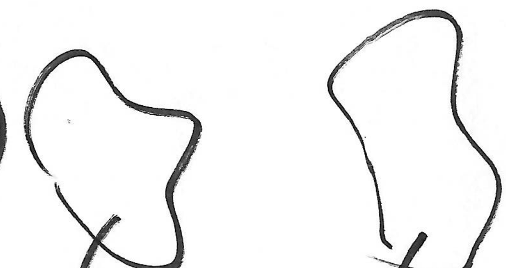
$$\left[\frac{c^2}{\omega^2} \vec{\nabla} \times \vec{\nabla} \times \cancel{\vec{1}} - \overleftrightarrow{1} - \overleftrightarrow{\chi}\right] \underset{(\vec{r},\omega)}{\vec{E}} = \vec{P}$$

(6) $$\left[\frac{c^2}{\omega^2}\vec{\nabla}\times\vec{\nabla}\times - \overleftrightarrow{1} - \overleftrightarrow{\chi}\right]\cdot\overleftrightarrow{\Gamma}(\vec{r},\vec{r}\,',\omega) = \overleftrightarrow{1}\,\delta^{(3)}(\vec{r}-\vec{r}\,')$$

Green's dyadic.

(7) $$U = \frac{\text{Energy}}{\text{volume}} = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

(8) $$E_{12} = \frac{\hbar}{2}\int_{-\infty}^{+\infty}\frac{d\zeta}{2\pi}\,\mathrm{tr}\ln\left(1-\overleftrightarrow{\Gamma}_1\chi_1\overleftrightarrow{\Gamma}_2\chi_2\right)$$



$\omega \to i\zeta$

vector + space.

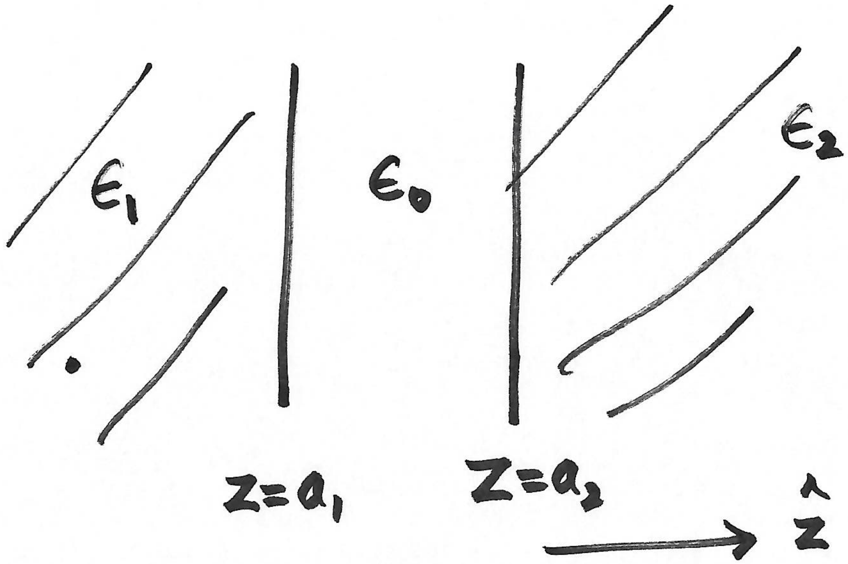
matrix function-kernel

χ_1

χ_2

two disjoint dielectric.

(9)



$$\chi_1 = \left(\frac{\epsilon_1}{\epsilon_0} - 1\right) \theta(a_1 - z)$$

$$\chi_2 = \left(\frac{\epsilon_2}{\epsilon_0} - 1\right) \theta(z - a_2)$$

$$\vec{k}_\perp = k_x \hat{i} + k_y \hat{j}.$$

$$\theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

(10) Translation symmetry in x & y. $\quad x \to k_x, \quad y \to k_y, \quad \vec{k}_\perp$

$$\overleftrightarrow{\Gamma}_i(\vec{r}, \vec{r}\,'; \omega) = \int \frac{d^2 k_\perp}{(2\pi)^2}\, e^{i \vec{k}_\perp \cdot (\vec{r} - \vec{r}\,')_\perp}\, \overleftrightarrow{\gamma}_i(z, z'; \underbrace{\vec{k}_\perp, \omega}_{\times})$$

reduced Green's dyadic.

11 $$\overleftrightarrow{\gamma}_i(z,z') = \begin{bmatrix} \frac{1}{\epsilon(z)}\frac{\partial}{\partial z}\frac{1}{\epsilon(z')}\frac{\partial}{\partial z'} g^H(z,z') & 0 & \frac{ik_\perp}{\epsilon(z)}\frac{1}{\epsilon(z')}\frac{\partial}{\partial z} g^H \\ 0 & -\zeta^2 g^E(z,z') & 0 \\ -\frac{ik_\perp}{\epsilon(z)}\frac{1}{\epsilon(z')}\frac{\partial}{\partial z'} g^H & 0 & \frac{k_\perp^2}{\epsilon(z)\epsilon(z')} g^H \end{bmatrix}$$

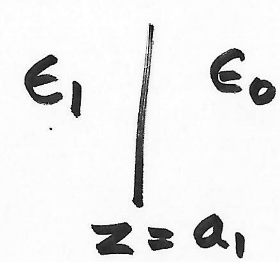
$\omega \to i\zeta$

$$-\left(\frac{\partial^2}{\partial z^2} - k_\perp^2 + \frac{\zeta^2}{c^2}\epsilon_i(z)\right) g^E(z,z') = \delta(z-z')$$

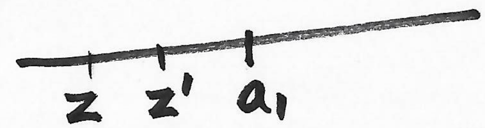
electric Green's function

$$-\left(\frac{\partial}{\partial z}\frac{1}{\epsilon(z)}\frac{\partial}{\partial z} - \frac{k_\perp^2}{\epsilon(z)} + \frac{\zeta^2}{c^2}\right) g^H(z,z') = \delta(z-z')$$

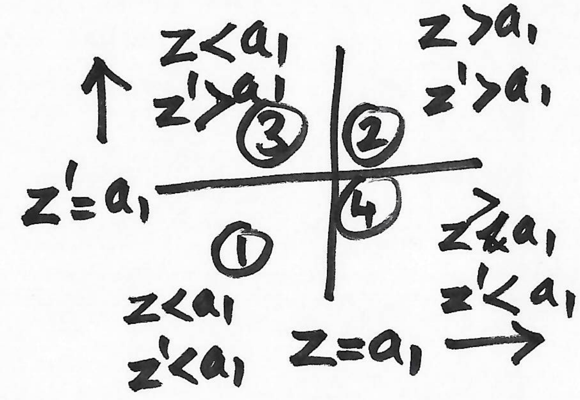
magnetic Green's function



(12) ~~One - plate~~ One-slab



Prachi's thesis



$$g^E_{③1}(z,z') = \frac{1}{K_1+K}\, e^{-K_1(a_1-z)}\, e^{-K(z'-a_1)}$$

$z<a_1$, $z'>a_1$ — slab

$$K_i^2 = k_\perp^2 + \frac{\varsigma^2}{c^2}\epsilon_i = \boxed{k_\perp^2 + \frac{\varsigma^2}{c^2}} + \frac{\varsigma^2}{c^2}(\epsilon_i - 1) = K^2 + \frac{\varsigma^2}{c^2}(\epsilon_i - 1)$$

(13) Second-slab

$$g^E_{④2}(z,z') = \frac{1}{K_2+K}\, e^{-K_2(z-a_2)}\, e^{-K(a_2-z')}$$

$$g^E_{①2}(z,z') = \frac{1}{K_2+K}\, e^{-K_2|z-z'|} + \frac{1}{K_2+K}\,\frac{K_2-K}{K_2+K}\, e^{-K_2(a_2-z)}\, e^{-K(a_2-z')}$$

$z<a_2$
$z'<a_2$

14 $$E_{12} = \frac{\hbar c}{2} \int_{-\infty}^{+\infty} \frac{d\zeta/c}{2\pi} \int \frac{d^2k_\perp}{(2\pi)^2} \ln(1-K)$$

$$K = \int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} dz' \; \overleftrightarrow{\gamma}_1(z,z')\, \chi_1(z')\, \overleftrightarrow{\gamma}_2(z',z)\, \chi_2(z)$$

$\downarrow$ tr — matrix

$$\chi_1 = \left(\frac{\epsilon_1}{\epsilon_0} - 1\right) \underline{\underline{\theta(a_1 - z')}}$$

$$= \left(\frac{\epsilon_1}{\epsilon_0} - 1\right)\left(\frac{\epsilon_2}{\epsilon_0} - 1\right) \int_{a_2}^{+\infty} dz \int_{-\infty}^{a_1} dz' \; \mathrm{tr}\left[\overleftrightarrow{\gamma}_1(z,z') \cdot \overleftrightarrow{\gamma}_2(z',z)\right]$$

$$\gamma_1 = \begin{bmatrix} H & 0 & H \\ 0 & \boxed{E} & 0 \\ H & 0 & H \end{bmatrix}$$

$$E \rightarrow -\zeta^2 g^E(z,z')$$

$$\gamma_1^H = \begin{bmatrix} H & H \\ H & H \end{bmatrix}$$

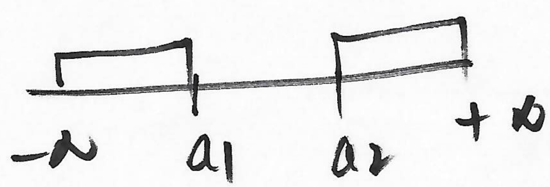
15 $$\ln(1-K) = \ln(1-K^E)(1-K^H)$$

(15)

$$K^E = \frac{\varsigma^4}{c^4}\left(\frac{\epsilon_1}{\epsilon_0}-1\right)\left(\frac{\epsilon_2}{\epsilon_0}-1\right)\int_{a_2}^{+\infty} dz \int_{-\infty}^{a_1} dz' \; g^E_{1\,④}(z,z')\; g^E_{2\,③}(z',z) \quad (10) \quad X_1$$

$$K_i^2 = k_\perp^2 + \frac{\varsigma^2}{c^2}\cancel{\epsilon}\,\epsilon_i$$

$$= K^2 + \frac{\varsigma^2}{c^2}(\epsilon_i - 1)$$



$$K^E = \frac{(K_1^2-K^2)}{K_1+K}\frac{(K_2^2-K^2)}{K_2+K}\int_{a_2}^{+\infty} dz \int_{-\infty}^{a_1} dz' \; e^{-\overset{K}{K_1}(z-a_1)}\, e^{-\overset{K_1}{K}(a_1-z')}$$

$$\times\, e^{-\underset{K}{K_2}(\overset{a_2-z'}{\cancel{z'-a_2}})}\, e^{-\overset{K_2}{K}(z-a_2)}$$

$$= (K_1-K)(K_2-K)\, e^{-Ka_1+Ka_2}\, e^{+K_1 a_1}\, e^{-K_2 a_2} \int_{a_2}^{\infty} dz\, e^{-z(K_1+K)} \int_{-\infty}^{a_1} dz'\, e^{\overset{+}{-} z'(K_2+K)}$$

?

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$$K^E = \left(\frac{\kappa_1-\kappa}{\kappa_1+\kappa}\right)\left(\frac{\kappa_2-\kappa}{\kappa_2+\kappa}\right) e^{-\kappa(a_1-a_2)} e^{\kappa_1 a_1 - \kappa_2 a_2} e^{-a_2(\kappa_1+\kappa)} e^{+a_1(\kappa_2+\kappa)}$$

$a_2 - a_1 = a.$

$$K^E = \left(\frac{\kappa_1-\kappa}{\kappa_1+\kappa}\right)\left(\frac{\kappa_2-\kappa}{\kappa_2+\kappa}\right) e^{+\kappa a} e^{-\kappa a} e^{\kappa_1 a_1 - \kappa_2 a_2 - \kappa_1 a_2 + \kappa_2 a_1}$$

$$K^E = (\kappa_1-\kappa)(\kappa_2-\kappa)\, e^{-\kappa(a_2-a_1)} e^{-\kappa_1 a_1 + \kappa_2 a_2} \int_{a_2}^{\infty} dz\, e^{-z(\kappa_2+\kappa)} \int_{-\infty}^{a_1} dz'\, e^{+z'(\kappa_1+\kappa)}$$

(17) $$K^E = \frac{\kappa_1-\kappa}{\kappa_1+\kappa}\,\frac{\kappa_2 \overset{-}{\cancel{+}} \kappa}{\kappa_2+\kappa}\, e^{-\kappa a}\, e^{-\kappa_1 a_1 + \kappa_2 a_2}\, e^{-a_2(\kappa_2+\kappa)}\, e^{a_1(\kappa_1+\kappa)}$$

$$= \frac{\kappa_1-\kappa}{\kappa_1+\kappa}\,\frac{\kappa_2 \overset{-}{\cancel{+}} \kappa}{\kappa_2+\kappa}\, e^{-2\kappa a}.$$

$$\bar{\kappa}_i = \frac{\kappa_1}{\epsilon_i}$$

Todo:

(18) $$K^H = \frac{\bar{\kappa}_1-\kappa}{\bar{\kappa}_1+\kappa}\,\frac{\bar{\kappa}_2-\kappa}{\bar{\kappa}_2+\kappa}\, e^{-2\kappa a}.$$

(19) $$\frac{E_{12}}{L_x L_y} = \frac{\hbar c}{2}\int_{-\infty}^{+\infty}\frac{d\xi/c}{2\pi}\int\frac{d^2k_\perp}{(2\pi)^2}\left[\ln\left(1-K^E\right)+\ln\left(1 \overset{-}{\cancel{+}} K^H\right)\right]$$

Lifshitz formula.

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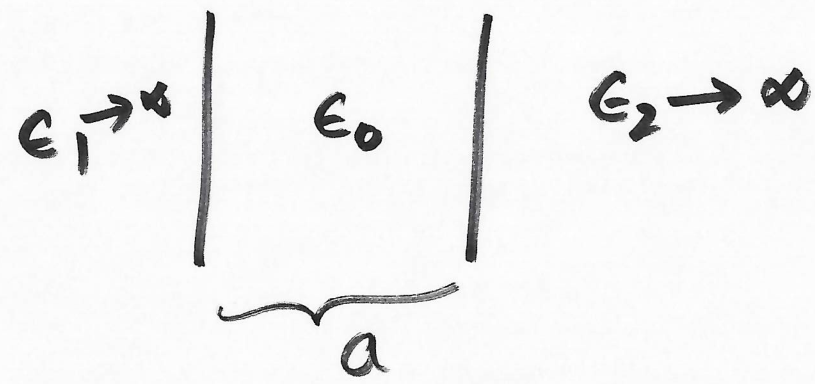

$$K_i^2 = k_\perp^2 + \frac{\xi^2}{c^2}\epsilon_i$$

$$\frac{K_1 - K}{K_1 + K} \to 1$$

$$\frac{\bar{K}_1 - K}{\bar{K}_1 + K} \to -1 .$$

$$\frac{E_{12}}{L_x L_y} = 2\,\frac{\hbar c}{2}\int_{-\infty}^{+\infty}\frac{d\xi/c}{2\pi}\int_{-\infty}^{+\infty}\frac{d^2k_\perp}{(2\pi)^2}\,\ln\left(1 - e^{-2Ka}\right)$$

$$K^2 = k_\perp^2 + \frac{\xi^2}{c^2}.$$

$$= k_x^2 + k_y^2 + \frac{\xi^2}{c^2}.$$

(21)

$$\frac{E_{12}}{L_x L_y} = \frac{\hbar c}{(2\pi)^3} \int d^3\kappa \, \ln\left(1 - e^{-2\kappa a}\right)$$

$$= \frac{\hbar c}{8\pi^3} 4\pi \int_0^\infty \kappa^2 d\kappa \, \ln\left(1 - e^{-2\kappa a}\right)$$

$x = 2\kappa a$

$$= \frac{\hbar c}{2\pi^2} \frac{1}{(2a)^3} \int_0^\infty x^2 dx \, \ln\left(1 - e^{-x}\right)$$

$$= \frac{\hbar c}{16\pi^2 a^3} \frac{1}{3} \int_0^\infty dx \left(\frac{d}{dx} x^3\right) \ln\left(1 - e^{-x}\right)$$

$$= \frac{\hbar c}{48\pi^2 a^3} \left[\underbrace{\int_0^\infty dx \frac{d}{dx}\left\{x^3 \ln(1-e^{-x})\right\}}_{=0} - \int_0^\infty dx \, x^3 \frac{e^{-x}}{1-e^{-x}} \right]$$

22 $$\frac{\bar{E}_{12}}{L_x L_y} = -\frac{\hbar c}{48\pi^2 a^3} \underbrace{\int_0^\infty dx \frac{x^3}{e^x - 1}}_{\zeta(4)\,\Gamma(4)}$$

$$\zeta(4) = \frac{\pi^4}{90}$$

$$\Gamma(4) = 3!$$

$$= -\frac{\hbar c}{48\pi^2 a^3} \frac{\pi^4}{90} 6$$

$$= -\frac{\pi^2}{720} \frac{\hbar c}{a^3}$$

✓