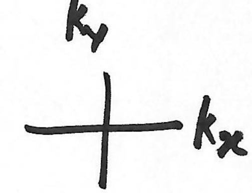
1 $\vec{\nabla} \times \vec{E} = i\omega \vec{B}$

$\vec{\nabla} \times \vec{H} = -i\omega(\vec{D} + \vec{P})$

$\vec{D} = \epsilon \vec{E}$

$\vec{B} = \mu \vec{H}$



Planar

$\vec{k}_\perp \parallel \hat{x}\, k_x$

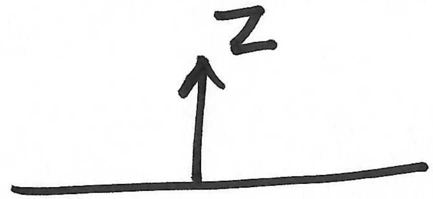
2 $\vec{\nabla} = \vec{\nabla}_\perp + \hat{z}\frac{\partial}{\partial z}$

$= i\vec{k}_\perp + \hat{z}\frac{\partial}{\partial z}$

~~3 $\hat{z}\cdot\vec{\nabla}\times\vec{E} = i\omega\,\hat{z}\cdot\vec{B}$~~

$\hat{z}\,,\; i\vec{k}_\perp\,,\; \hat{z}\times i\vec{k}_\perp$

3



$$\vec{E} = \hat{z}(\hat{z}\cdot\vec{E}) + \frac{i\vec{k}_\perp(i\vec{k}_\perp\cdot\vec{E})}{(ik_\perp)^2} + \frac{(\hat{z}\times i\vec{k}_\perp)\;\hat{z}\times i\vec{k}_\perp\cdot\vec{E}}{(ik_\perp)^2}.$$

$$= \hat{z}\,E_z + i\hat{k}_\perp E_u + (\hat{z}\times i\hat{k}_\perp)\,E_v(z, k_\perp)$$

ω

4 $$\vec{\nabla} \times \vec{E} = \left(i\vec{k}_\perp + \hat{z}\frac{\partial}{\partial z} \right) \times \left[\hat{z} E_z + i\hat{k}_\perp E_u + (\hat{z} \times i\hat{k}_\perp) E_v \right]$$

$$= -(\hat{z} \times i\hat{k}_\perp) \overset{k_\perp \swarrow}{E_z} - \hat{z} k_\perp E_v + \hat{z} \times i\hat{k}_\perp \frac{\partial}{\partial z} E_u - i\hat{k}_\perp \frac{\partial}{\partial z} E_v$$

$$= -\hat{z} k_\perp E_v - i\hat{k}_\perp \frac{\partial}{\partial z} E_v + (\hat{z} \times i\hat{k}_\perp) \left\{ \frac{\partial}{\partial z} E_u - \overset{k_\perp \swarrow}{E_z} \right\}$$

5 $$i\omega\vec{B} = \hat{z}\, i\omega B_z + i\hat{k}_\perp\, i\omega B_u + (\hat{z} \times i\hat{k}_\perp)\, i\omega B_v$$

6 $\hat{z}$: $\quad -k_\perp E_v = i\omega B_z$

$i\hat{k}_\perp$: $\quad -\frac{\partial}{\partial z} E_v = i\omega B_u$

$\hat{z} \times i\hat{k}_\perp$: $\quad \frac{\partial}{\partial z} E_u - \overset{k_\perp \swarrow}{E_z} = i\omega B_v$

7

$$-k_\perp E_v = i\omega B_z \qquad \text{(i)}$$

$$-\frac{\partial}{\partial z} E_v = i\omega B_u \qquad \text{(ii)}$$

$$\boxed{\frac{\partial}{\partial z} E_u - \overset{k_\perp}{E_z} = i\omega B_v \qquad \text{(iii)}}$$

$$\text{(iv)} \quad -k_\perp H_v = -i\omega \left[D_z + P_z \right]$$

$$\text{(v)} \quad -\frac{\partial}{\partial z} H_v = -i\omega \left[D_u + P_u \right]$$

$$\text{(vi)} \quad \frac{\partial}{\partial z} H_u - \overset{k_\perp}{H_z} = -i\omega \left[D_v + P_v \right]$$

8 (iii) $\cancel{\frac{\partial^2}{\partial z^2} E_u - \frac{\partial}{\partial z} E_z = i\omega \frac{\partial}{\partial z} H_v}$

$$\frac{\partial^2}{\partial z^2}$$

9 (v)

$$-\frac{\partial}{\partial z} H_v = -i\omega\epsilon E_u - i\omega P_u.$$

$$\frac{1}{i\omega}\frac{1}{\epsilon}\frac{\partial}{\partial z} H_v = E_u + \frac{P_u}{\epsilon}$$

(10) $$\frac{1}{i\omega}\frac{\partial}{\partial z}\frac{1}{\epsilon}\frac{\partial}{\partial z}H_v = \frac{\partial}{\partial z}E_u + \frac{\partial}{\partial z}\frac{P_u}{\epsilon}$$

using (III) above.

$k_\perp$

$$\frac{1}{i\omega}\frac{\partial}{\partial z}\frac{1}{\epsilon}\frac{\partial}{\partial z}H_v = E_z + i\omega H_v + \frac{\partial}{\partial z}\frac{P_u}{\epsilon}$$

using (IV) above.

$$\frac{1}{i\omega}\frac{\partial}{\partial z}\frac{1}{\epsilon}\frac{\partial}{\partial z}H_v = k_\perp\left(\frac{k_\perp H_v}{i\omega} - P_z\right)\frac{1}{\epsilon} + i\omega H_v + \frac{\partial}{\partial z}\frac{P_u}{\epsilon}$$

$$\frac{\partial}{\partial z}\frac{1}{\epsilon}\frac{\partial}{\partial z}H_v - \frac{k_\perp^2}{\epsilon}H_v + \omega^2 H_v = -\frac{k_\perp}{\epsilon}i\omega P_z + i\omega\frac{\partial}{\partial z}\frac{P_u}{\epsilon}.$$

$$-\left[\frac{\partial}{\partial z}\frac{1}{\epsilon}\frac{\partial}{\partial z} - \frac{k_\perp^2}{\epsilon} + \omega^2\right]H_v(z,k_\perp,\omega) = i\omega\left[\frac{k_\perp}{\epsilon}P_z - \frac{\partial}{\partial z}\frac{P_u}{\epsilon}\right]$$

(11) $$-\left[\frac{\partial}{\partial z}\frac{1}{\epsilon}\frac{\partial}{\partial z} - \frac{k_\perp^2}{\epsilon} + \omega^2\right] g^H(z,z') = \delta(z-z')$$

$$g^H(z,z';k_\perp,\omega)$$

H-mode

(12)
$$\frac{\partial}{\partial z} E_u - k_\perp E_z = i\omega H_v \qquad \text{— (i)}$$

$$\frac{-k_\perp}{-i\omega\epsilon} H_v = E_z + \frac{P_z}{\epsilon} \qquad \text{— (ii)}$$

$$-\frac{\partial}{\partial z} H_v = -i\omega\left[\epsilon E_u + P_u\right] \qquad \text{— (iii)}$$

(13) Using (10) and (11)

$$H_v(z) = \int dz'\, g^H(z,z')\; i\omega\left[\frac{k_\perp}{\epsilon}P_z - \frac{\partial}{\partial z}\frac{P_u}{\epsilon}\right]$$

14 Using 13 in 12 -(ii)

$$E_z = \frac{k_\perp}{i\omega}\frac{1}{\epsilon} H_v - \frac{P_z}{\epsilon}$$

$$= \frac{k_\perp}{\cancel{i\omega}} \frac{1}{\epsilon(z)} \int dz' \, g^H(z,z') \, \cancel{i\omega} \left[\frac{k_\perp}{\epsilon(z')} P_z - \frac{\partial}{\partial z'} \frac{P_u}{\epsilon(z')} \right] - \frac{P_z}{\epsilon(z)}$$

$$= \int dz' \, \frac{k_\perp^2}{\epsilon(z)\,\epsilon(z')} \, g^H(z,z') \, P_z(z')$$

$$+ \int dz' \, \frac{k_\perp}{\epsilon(z)} \frac{1}{\epsilon(z')} \left\{ \frac{\partial}{\partial z'} g^H(z,z') \right\} P_u(z')$$

$$- \int dz' \, \frac{\delta(z-z')}{\epsilon(z)} \, P_z(z')$$

$$\cancel{E_z = \int dz' \left[\frac{k_\perp}{\epsilon(z)} \frac{1}{\epsilon(z')} \frac{\partial}{\partial z'} + k_\perp \right.}$$

15 Using 13 in 12 -(iii)

E_u
H_v
E_z

$$\frac{1}{i\omega}\frac{\partial}{\partial z} H_v = \epsilon E_u + P_u$$

$$E_u = \frac{1}{i\omega}\frac{1}{\epsilon}\frac{\partial}{\partial z} H_v - \frac{P_u}{\epsilon}$$

$$E_u = \cancel{\frac{1}{i\omega}}\frac{1}{\epsilon(z)}\frac{\partial}{\partial z}\int dz'\, g^H(z,z')\, \cancel{i\omega}\left\{\frac{k_\perp}{\epsilon(z')} P_z - \frac{\partial}{\partial z'}\frac{P_u}{\epsilon(z')}\right\} - \frac{P_u}{\epsilon}$$

$$= \int dz' \left\{\frac{k_\perp}{\epsilon(z')}\frac{1}{\epsilon(z)}\frac{\partial}{\partial z} g^H(z,z')\right\} P_z$$

$$+ \int dz' \left\{\frac{1}{\epsilon(z)}\frac{\partial}{\partial z}\frac{1}{\epsilon(z')}\frac{\partial}{\partial z'} g^H(z,z')\right\} P_u \checkmark$$

$$- \int dz' \frac{\delta(z-z')}{\epsilon} P_u(z')$$

$$\begin{bmatrix} E_u(z) \\ E\cancel{H}_v(z) \\ E_z(z) \end{bmatrix} = \int dz' \begin{bmatrix} \frac{1}{\epsilon(z)}\frac{\partial}{\partial z}\frac{1}{\epsilon(z')}\frac{\partial}{\partial z'} g^H & 0 & \frac{k_\perp}{\epsilon(z')}\frac{1}{\epsilon(z)}\frac{\partial}{\partial z} g^H \\ & & \\ \frac{k_\perp}{\epsilon(z)}\frac{1}{\epsilon(z')}\frac{\partial}{\partial z'} g^H & 0 & \frac{k_\perp^2}{\epsilon(z)\epsilon(z')} g^H \end{bmatrix} \begin{bmatrix} P_u(z') \\ P_v(z') \\ P_z(z') \end{bmatrix}$$

$$+ \int dz' \begin{bmatrix} -\frac{\delta(z-z')}{\epsilon(z)} & 0 & 0 \\ 0 & 0 & \\ 0 & 0 & -\frac{\delta(z-z')}{\epsilon(z)} \end{bmatrix} \vec{P}$$

factor of i

$\rightarrow$ disjoint conf. $= 0$

$$\vec{E}(z) = \int dz' \overleftrightarrow{\Gamma}(z,z') \cdot \vec{P}(z')$$