Electromagnetism 5

I Work done to move a point charge

1

{Also refer electromagnetism 9}

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{l}$$

$$= q \int_a^b \vec{E} \cdot d\vec{l}$$

$$= q \int_a^b d\vec{l} \cdot \vec{\nabla} \phi$$

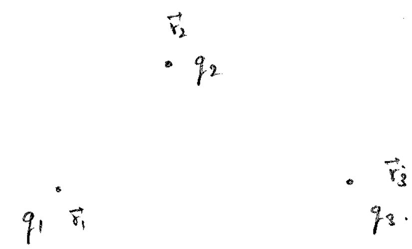
$$= q \left[\phi(b) - \phi(a) \right]$$

2 Reference point can be chosen to be 'a'.

Or maybe infinity.

II Potential energy

1. Potential energy is the work done to assemble a set of charges. For three charges it would be



2. $$W = \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|} + \frac{q_2 q_3}{|\vec{r}_2 - \vec{r}_3|} + \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|}$$

3. For 'n' charges.

$$W = \frac{1}{2} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$$

$$= \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} - \underbrace{\sum_i \frac{q_i^2}{|\vec{r}_i - \vec{r}_i|}}_{\text{"self-action"}}$$

"self-action" → work done to create the point charge.

$$= \frac{1}{2} \sum_i q_i \, \phi(\vec{r}_i) - \text{"self-action"}$$

III Potential energy due to a charge distribution

① $$W = \frac{1}{2}\int d^3r \; \rho(\vec{r})\,\phi(\vec{r})$$ — "self-action" work done to create the point charges.

$$= \frac{1}{2}\int d^3r \; \rho(\vec{r})\,\phi(\vec{r}) \qquad \longleftarrow \; ?$$

② $$W = \frac{1}{8\pi}\int d^3r \left[\vec{\nabla}\cdot\vec{E}\right]\phi \qquad (\text{using } \vec{\nabla}\cdot\vec{E} = 4\pi\rho)$$

$$= \frac{1}{8\pi}\int d^3r \left[\vec{\nabla}\cdot(\vec{E}\phi) - \vec{E}\cdot\vec{\nabla}\phi\right]$$

$$= -\frac{1}{8\pi}\int d^3r \; \vec{E}\cdot\vec{\nabla}\phi + \frac{1}{8\pi}\oint_S d\vec{a}\cdot[\vec{E}\phi]$$

$$= +\frac{1}{8\pi}\int d^3r \; E^2 + \frac{1}{8\pi}\oint_S d\vec{a}\cdot[\vec{E}\phi]$$

③ If $[\vec{E}\phi]$ is zero on the surface

$$W = \frac{1}{8\pi}\int d^3r \; E^2$$

IV Conductors

① Electrons in a conductor are free to move.

② Properties :

(a) $\vec{E} = 0$ inside a conductor.

→ if not the electrons will move in such a way that $\vec{E} = 0$ after movement.

(b) $\rho = 0$ inside a conductor.

→ $\vec{\nabla} \cdot \vec{E} = 4\pi \rho$

(c) free charge resides on the surface.

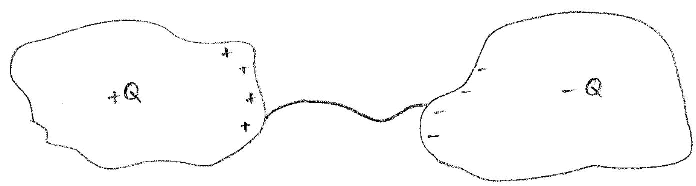
(d) a conductor is an equipotential

→ $\phi(b) - \phi(a) = -\int_a^b \vec{E} \cdot d\vec{l} = 0$

(e) $\vec{E}$ is perpendicular to surface $(\vec{E}_{\parallel} = 0)$.

(f) a grounded conductor means $\phi = 0$.

V Capacitors

① 

② $V = \phi_+ - \phi_-$

$$= - \int_-^+ \vec{E} \cdot d\vec{l}$$

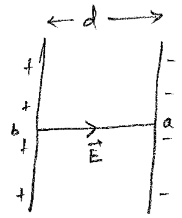
← unique for a given pair of conductors.

③ $$\vec{E}(\vec{r}) = \int ds_1 \frac{\sigma_1}{|\vec{r} - \vec{s}_1|^2} + \int ds_2 \frac{\sigma_2}{|\vec{r} - \vec{s}_2|^2}$$

④ $V = \dfrac{Q}{C}$ → capacitance

C depends only on the geometry of the surface of conductors.

(5) Parallel plate capacitor :



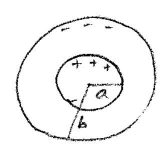
$$EA = 4\pi \sigma A$$
$$E = 4\pi \sigma$$

$$V = -\int_a^b \vec{E} \cdot d\vec{l} \quad - \quad Ed$$

$$V = 4\pi \sigma d$$
$$= 4\pi \frac{Qd}{A}$$

$$\Rightarrow \quad C = \frac{A}{4\pi d}$$

(6) Capacitance of two concentric spheres :



$$V = -\int_b^a \vec{E} \cdot d\vec{l}$$
$$= \int_b^a \frac{Q}{r^2}\, dr$$
$$= \frac{Q}{r}(-1)\Big|_b^a = Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$\Rightarrow \quad C = \frac{ab}{a-b}$$