Homework No. 2

1. Show that $\vec{\nabla}\cdot(\vec{x}\times\vec{A}) = \vec{x}\cdot(\vec{\nabla}\times\vec{A})$.

(Use tensor notation).

2. Verify the following:

(i) $Tr(A) = A^i{}_i$

(ii) $\det(A) = \epsilon_{i_1 i_2 \dots i_n} A^{i_1}{}_1 \, A^{i_2}{}_2 \dots A^{i_n}{}_n$

$$= \frac{1}{n!} \epsilon_{i_1 i_2 \dots i_n} \epsilon^{i_1' i_2' \dots i_n'} A^{i_1}{}_{i_1'} A^{i_2}{}_{i_2'} \dots A^{i_n}{}_{i_n'}$$

where 'n' is the dimension of the matrix 'A'.

3. Prove that

$$[\det. L]^2 = 1$$

where L is any Lorentz matrix.

You <u>need not</u> use index notation.

4. Prove that any orthogonal matrix R satisfying

$$R R^T = 1$$

in N-dimensions has $\frac{N(N-1)}{2}$ independent variables.

(5) (i) Evaluate $R_x R_y$

(ii) Evaluate $R_y R_x$

Thus conclude that $R_x R_y \neq R_y R_x$.

(iii) Evaluate $[R_x, R_y] \equiv R_x R_y - R_y R_x$

(6) Problem from Hughston & Tod's book : Prove

(i) if p^a is time-like

and $p^a S_a = 0$

then S_a is space-like.

(ii) If p^a and Q^a are time-like

and $p^a Q_a > 0$

then either both are future-pointing or both are past-pointing.

(7)
$$L_x = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Find L_x^{-1}.

(8) Work on : Prob. 12.7 and 12.8, Griffiths (time dilation)

(9) Work on : Prob. 24.3, 12.4, 12.15, 12.19 (Velocity addition)