Homework No. 3

① Work on : Griffiths 12.20, 12.21 (based on space-time concepts)

② Based on 4-velocity

work on : Griffiths 12.24 to 12.27

eqn. 12.40 $\vec{\eta} = \dfrac{\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}}$ → ordinary velocity

↓

proper velocity

eqn. 12.34 → refer prob. 12.19

eqn. 12.37 → $d\tau = \sqrt{1-\frac{v^2}{c^2}}\ dt$

③ 4-momentum

Work on 12.29, 12.30. (Griffiths)

④ 4-acceleration

Work on Griffiths 12.38 a, b, c. Skip part (d).

④ Non-relativistic limits:

Show that for $\beta < 1$

(i) $\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{1}{2}\beta^2 + O(\beta^4)$

(ii) $\beta\gamma = \beta + O(\beta^3)$

(iii) $L_x = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} = \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix} + O(\beta^2)$

→ Do we recover Galilean transformation for $\beta << 1$?

→ Next take the limit $c \to \infty$. Do we recover Galilean transformation now?

(iv) $u^\alpha = (c\gamma, c\gamma\beta) = (c, \vec{v}) + O(\beta^2)$

(v) $\frac{\vec{p}}{c} = \frac{1}{c}\frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m\vec{v}}{c} + O(\beta^3)$

(vi) $\frac{E}{c^2} = \frac{1}{c^2}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{mc^2}{c^2} + \frac{1}{c^2}\frac{1}{2}mv^2 + O(\beta^4)$

(vii) Is there any non-relativistic limits for photons?