Homework No. ⑥

① 4- acceleration

(a) Show that $a^\mu = \gamma\left(c\frac{d\gamma}{dt} , \vec{v}\frac{d\gamma}{dt} + \vec{a}\gamma \right)$

(b) Show that in the instantaneous rest frame of particle ($\vec{v} = 0$)

$$a^\mu = (0 , \vec{a}_0)$$

where $\vec{a}_0$ is the 3-acceleration in the instantaneous rest frame.

(c) Prove that 4-acceleration a^μ is a space-like Lorentz vector. (Hint : Evaluate $a^\mu a_\mu$ in the instantaneous rest frame of the particle.)

(d) Using the expression for a^μ in (a) in a general inertial frame show that

(i) $$a^\mu a_\mu = -\left[\gamma^4 a^2 + \gamma^6 \frac{(\vec{v}\cdot\vec{a})^2}{c^2} \right]$$

(ii) $$a^\mu a_\mu = -\gamma^6 \left[a^2 - \frac{(\vec{v}\times\vec{a})^2}{c^2} \right]$$

(e) Using (b) and (d) show that

(i) when $\vec{a}$ parallel to $\vec{v}$ in a particular frame (linear motion)

$$|\vec{a}_0| = \gamma^3 |\vec{a}|$$

(ii) when $\vec{a}$ perpendicular to $\vec{v}$ in a particular frame (circular motion)

$$|\vec{a}_0| = \gamma^2 |\vec{a}|$$

② Work on problem ① in HW # 4

③ Work on the following problems from the book on relativity : special, general & cosmological by W. Rindler.

5.4 , 5.4 , 5.19(i) (velocity , acceleration)

5.11 , 5.12 (space-time concepts)

6.1 , 6.2 (Energy , momentum)

6.13 , 6.15 (collisions).

④ We saw

$$\vec{F} = \gamma m \vec{a} + \vec{v} \frac{(\vec{F} \cdot \vec{v})}{c^2}$$

(a) Show that $\vec{F}$, $\vec{a}$ and $\vec{v}$ are co-planar.

(b) Show that $\vec{F}$ and $\vec{a}$ are in general not parallel.