Homework No. ⑥

① Given

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ +E_x & 0 & -B_z & +B_y \\ +E_y & +B_z & 0 & -B_x \\ +E_z & -B_y & +B_x & 0 \end{bmatrix}$$

(i) Show that

$$F_{\alpha\beta} = g_{\alpha\mu} F^{\mu\nu} g_{\nu\beta}$$

$$= \begin{bmatrix} 0 & +E_x & +E_y & +E_z \\ -E_x & 0 & -B_z & +B_y \\ -E_y & +B_z & 0 & -B_x \\ -E_z & -B_y & +B_x & 0 \end{bmatrix}$$

(ii) Show that

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$= \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ +B_x & 0 & +E_z & -E_y \\ +B_y & -E_z & 0 & +E_x \\ +B_z & E_y & -E_x & 0 \end{bmatrix}$$

(iii) Evaluate

$$\tilde{F}_{\alpha\beta} = g_{\mu\alpha} \tilde{F}^{\alpha\beta} g_{\beta\nu}$$

(2) Show that

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\mu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

reproduces the four Maxwell's equations.

(Hint : $F^{ij} = -\epsilon^{ijk} B_k$)

(3) Show that

(i) $F^{0\alpha} u_\alpha = \gamma (\vec{E} \cdot \vec{v})$

(ii) $F^{i\alpha} u_\alpha = c\gamma \left[E^i + \epsilon^{ijk} \frac{v^j}{c} B_k \right]$

(4) Work on Rindler problem 6.21

→ rest mass preserving means $\frac{dm}{dt} = 0$.

→ Hint: $\vec{F} \cdot \vec{v} = \frac{dE}{dt} \Rightarrow \vec{F} \cdot d\vec{r} = dE \longrightarrow$ (eqn. 6.45)

Integrate eqn. 6.45. Use $d\,r^2 = \vec{r} \cdot d\vec{r}$.