

Midterm Exam No. 01 (Spring 2014)

PHYS 520B: Electromagnetic Theory

Date: 2014 Feb 21

1. **(20 points.)** A charged particle initially moving with constant speed v enters a region of magnetic field \mathbf{B} pointing into the page. It is deflected as shown in Fig. 1.

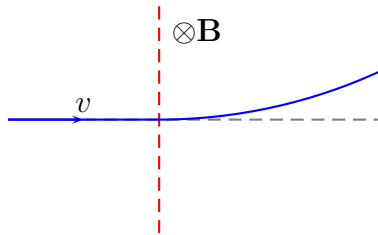


Figure 1: Problem 1

- (a) Is the charge on the particle positive or negative?
- (b) What curve characterizes the path of the deflected particle?
2. **(20 points.)** A steady current I flows through a wire shown in Fig. 2. Find the magnitude and direction of magnetic field at point P .

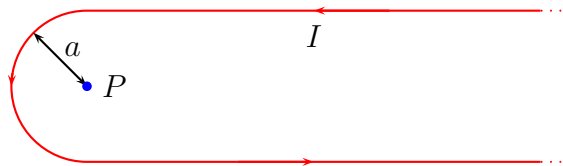


Figure 2: Problem 2

You are given the magnitude of the magnetic field due to an infinite length of wire at distance ρ , and a circular loop of wire of radius R at the center of loop, to be

$$B_{\infty\text{-wire}} = \frac{\mu_0 I}{2\pi\rho} \quad B_{\text{loop}} = \frac{\mu_0 I}{2R}. \quad (1)$$

3. **(20 points.)** The spherical harmonics are given by

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l+m)!}{(l-m)!}} \left(\frac{e^{i\phi}}{\sin \theta} \right)^m \left(\frac{d}{d \cos \theta} \right)^{l-m} \frac{(\cos^2 \theta - 1)^l}{2^l l!}. \quad (2)$$

Express $Y_{l,-l}(\theta, \phi)$ in simplified form.

4. **(20 points.)** A homogeneous magnetic field \mathbf{B} is characterized by the vector potential

$$\mathbf{A} = \frac{1}{2} \mathbf{B} \times \mathbf{r}. \quad (3)$$

- (a) Evaluate $\nabla \times \mathbf{A}$.
 - (b) Evaluate $\nabla \cdot \mathbf{A}$.
 - (c) Is this construction unique? (Hint: Remember the freedom of gauge transformation.)
 - (d) Now, for the case of $\mathbf{B} = (0, 0, B)$, pointing in the z direction, show that $\mathbf{A} = (0, Bx, 0)$ is a solution. Find another solution.
5. **(20 points.)** Is the relation

$$(\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} = \nabla \left(\frac{\boldsymbol{\mu} \cdot \mathbf{r}}{r^3} \right) \quad (4)$$

correct? ($\boldsymbol{\mu}$ is a position independent vector.)

- (a) If yes, prove the relation.
- (b) If not, disprove the relation.