MT-03 solutions (PHYS-205A, Fall 2014)

MT-03, prob 1

$\Delta h = h_a - h_c$.

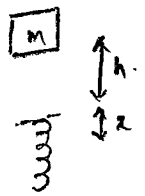
$$mg\,\Delta h - W = \frac{1}{2} m v_c^2$$

$$W = mg\,\Delta h - \frac{1}{2} m v_c^2$$

$$= 3.00 \times 9.8 \times (7.00 - 2.00) - \frac{1}{2} 3.00 \times 8.0^2$$

$$= 147 - 96 = 51 \text{ J}$$

MT-03, prob 2



$$mg(h+z) = \frac{1}{2} k x^2.$$

$$h = \frac{k x^2}{2mg} - x.$$

$$= \frac{1960 \times (0.2)^2}{2 \times 10.0 \times 9.8} - 0.2$$

$$= 0.2 \text{ m}$$

MT-03, prob 3

$$U = x^2 - x^3$$

$$F = -\frac{d}{dx} U = -2x + 3x^2$$

$F = 0 \Rightarrow x(3x-2) = 0$

$\Rightarrow x = 0$ and $x = \frac{2}{3}$ m.

MT-03, prob 4

A – Arrow
B – Person

$$M_B \cancel{V_{Bi}}^{=0} + M_A \cancel{V_{Ai}}^{=0} = M_B V_{Bf,x} + M_A V_{Af,x}$$

$$V_{Bf,x} = -\frac{M_A}{M_B} V_{Af,x}$$

$$= -\frac{M_A}{M_B} V_{Af} \cos\theta$$

$$= -\frac{0.030}{60.0} \times 80 \times \cos 60$$

$$= -0.02 \frac{m}{s} = -2 \frac{cm}{s}.$$

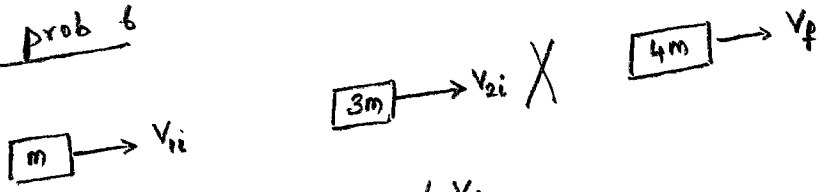
MT-03, prob 5

$$m_x V_x + m_y V_y = 0 \Rightarrow m_x V_x = -m_y V_y$$

$$m_x = \frac{3}{5} m_y$$

$$\frac{K_x}{K_y} = \frac{\frac{1}{2} m_x V_x^2}{\frac{1}{2} m_y V_y^2} = \frac{m_y}{m_x} = \frac{5}{3}$$

MT-03, prob 6

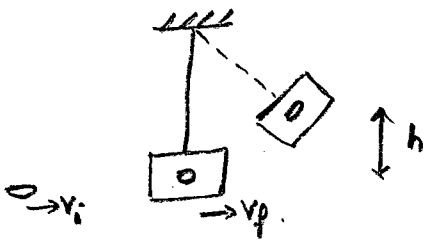


$$\cancel{m} V_{1i} + 3\cancel{m} V_{2i} = 4\cancel{m} V_f$$

$$3.90 \frac{m}{s} + 3 \times 1.95 \frac{m}{s} = 4 V_f$$

$$V_f = 2.44 \frac{m}{s}$$

MT-03, prob 7



$$m_B v_i = (m_B + M) v_f$$

$$v_i = \frac{(m_B + M)}{m_B} v_f$$

$$\frac{1}{2}(m_B + M) v_f^2 = (m_B + M) g h.$$

$$v_f = \sqrt{2gh}$$

$$v_i = \frac{(m_B + M)}{m_B} \sqrt{2gh} = \frac{0.005 + 2.5}{0.005} \times \sqrt{2 \times 9.8 \times 4 \times 10^{-2}}$$

$$= 443.6 \ \frac{m}{s}$$

MT-03, prob 8

$$\frac{dm}{dx} = \lambda = ax.$$

$$dm = ax\,dx.$$

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$= \frac{\int_0^L x\, a x\, dx}{\int_0^L a x\, dx}$$

$$= \frac{\int_0^L x^2 dx}{\int_0^L x\, dx}$$

$$= \frac{L^3/3}{L^2/2} = \frac{2}{3} L.$$