Homework No. 09 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2014 Nov 21, 4:00 PM

1. (**10 points.**) Interaction energy of a dipole $\mathbf{d}$ with an electric field $\mathbf{E}$ is
$$U = -\mathbf{d} \cdot \mathbf{E} = -dE\cos\theta. \tag{1}$$
The torque on the dipole due to the electric field is
$$\boldsymbol{\tau} = \mathbf{d} \times \mathbf{E}. \tag{2}$$
Force is a manifestation of the systems tendency to minimize its energy, and in this spirit torque is defined as,
$$\tau = -\frac{\partial}{\partial\theta} U = -dE\sin\theta. \tag{3}$$
Show that there is no inconsistency, in sign, between the two definitions of torque.

2. (**50 points.**) Consider the differential equation
$$\left[-\frac{\partial}{\partial z}\varepsilon(z)\frac{\partial}{\partial z} + \varepsilon(z)k_\perp^2 \right] g_\varepsilon(z,z') = \delta(z-z'), \tag{4}$$
for the case
$$\varepsilon(z) = \begin{cases} \varepsilon_2, & z < 0, \\ \varepsilon_1, & 0 < z, \end{cases} \tag{5}$$
satisfying the boundary conditions
$$g_\varepsilon(-\infty, z') = 0, \tag{6a}$$
$$g_\varepsilon(+\infty, z') = 0. \tag{6b}$$

   (a) Verify, by integrating Eq. (4) around $z = z'$, that the Green function satisfies the continuity conditions
$$g_\varepsilon(z,z')\Big|_{z=z'-\delta}^{z=z'+\delta} = 0, \tag{7a}$$
$$\varepsilon(z)\frac{\partial}{\partial z} g_\varepsilon(z,z')\Big|_{z=z'-\delta}^{z=z'+\delta} = -1. \tag{7b}$$

(b) Verify, by integrating Eq. (4) around $z = 0$, that the Green function satisfies the continuity conditions

$$g_\varepsilon(z, z')\Big|_{z=0-\delta}^{z=0+\delta} = 0, \tag{8a}$$

$$\varepsilon(z)\frac{\partial}{\partial z} g_\varepsilon(z, z')\Big|_{z=0-\delta}^{z=0+\delta} = 0. \tag{8b}$$

(c) For $z' < 0$, construct the solution in the form

$$g_\varepsilon(z, z') = \begin{cases} A_1 e^{k_\perp z} + B_1 e^{-k_\perp z}, & z < z' < 0, \\ C_1 e^{k_\perp z} + D_1 e^{-k_\perp z}, & z' < z < 0, \\ E_1 e^{k_\perp z} + F_1 e^{-k_\perp z}, & z' < 0 < z. \end{cases} \tag{9}$$

Determine the constants using the boundary conditions and continuity conditions.

(d) For $0 < z'$, construct the solution in the form

$$g_\varepsilon(z, z') = \begin{cases} A_2 e^{k_\perp z} + B_2 e^{-k_\perp z}, & z < 0 < z', \\ C_2 e^{k_\perp z} + D_2 e^{-k_\perp z}, & 0 < z < z', \\ E_2 e^{k_\perp z} + F_2 e^{-k_\perp z}, & 0 < z' < z. \end{cases} \tag{10}$$

Determine the constants using the boundary conditions and continuity conditions.

(e) Thus, find the solution

$$g_\varepsilon(z, z') = \begin{cases} \dfrac{1}{\varepsilon_2}\dfrac{1}{2k_\perp} e^{-k_\perp |z-z'|} + \dfrac{1}{\varepsilon_2}\dfrac{1}{2k_\perp}\left(\dfrac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}\right) e^{-k_\perp |z|} e^{-k_\perp |z'|}, & z' < 0, \\ \dfrac{1}{\varepsilon_1}\dfrac{1}{2k_\perp} e^{-k_\perp |z-z'|} + \dfrac{1}{\varepsilon_1}\dfrac{1}{2k_\perp}\left(\dfrac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\right) e^{-k_\perp |z|} e^{-k_\perp |z'|}, & 0 < z'. \end{cases} \tag{11}$$

3. (**10 points.**) A perfectly conducting plate is placed at $z = 0$ plane. A positive charge q is placed at $\mathbf{r} = d\,\hat{\mathbf{z}}$. Determine the direction and magnitude of electric field at $\mathbf{r} = d\,\hat{\mathbf{x}} + 2d\,\hat{\mathbf{z}}$.

4. (**10 points.**) A positive charge q and a negative charge q, a distance d apart from each other, are placed a distance $d/2$ away from a perfectly conducting plate. Determine the electrostatic force on the conductor?