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(1) We gave the conversions as.

$$\vec{D}_G = \sqrt{\frac{4\pi}{\epsilon_0}}\ \vec{D}_{SI} = \sqrt{4\pi}\ \vec{D}_{LH}$$

$$\vec{E}_G = \sqrt{4\pi\epsilon_0}\ \vec{E}_{SI} = \sqrt{4\pi}\ \vec{E}_{LH}$$

$$\vec{H}_G = \sqrt{4\pi\mu_0}\ \vec{H}_{SI} = \sqrt{4\pi}\ \vec{H}_{LH}$$

$$\vec{B}_G = \sqrt{\frac{4\pi}{\mu_0}}\ \vec{B}_{SI} = \sqrt{4\pi}\ \vec{B}_{LH}$$

$$\rho_G = \frac{1}{\sqrt{4\pi\epsilon_0}}\ \rho_{SI} = \frac{1}{\sqrt{4\pi}}\ \rho_{LH}$$

$$\vec{P}_G = \frac{1}{\sqrt{4\pi\epsilon_0}}\ \vec{P}_{SI} = \frac{1}{\sqrt{4\pi}}\ \vec{P}_{LH}$$

$$\vec{J}_G = \frac{1}{\sqrt{4\pi\epsilon_0}}\ \vec{J}_{SI} = \frac{1}{\sqrt{4\pi}}\ \vec{J}_{LH}$$

$$\vec{M}_G = \sqrt{\frac{\mu_0}{4\pi}}\ \vec{M}_{SI} = \frac{1}{\sqrt{4\pi}}\ \vec{M}_{LH}$$

Note: $\sqrt{\frac{4\pi}{\mu_0}} = c\sqrt{4\pi\epsilon_0}$

(2) The above equations let us write down the formulas from one system to the other. But, for relating numerical values we need to take care of the fact that

$$L_{SI}\ \lambda_1 = L_G \qquad \lambda_1 = 10^2$$

$$M_{SI}\ \lambda_2 = M_G \qquad \lambda_2 = 10^3$$

$$T_{SI} = T_G$$

(3) Let us consider the case of charge density

$$\rho_G = ?\ \frac{1}{\sqrt{4\pi\epsilon_0}}\ \rho_{SI}\ .$$

To this end we consider the electrostatic energy between two electrons,

$$U_{SI} = \frac{e_{SI}^2}{4\pi\epsilon_0\ r_{SI}} \qquad [U] = ML^2T^{-2}$$

$$U_G = \frac{e_G^2}{r_G} \qquad r_{SI}\,\lambda_1 = r_G$$

$$U_{SI}\,\lambda_1^2\,\lambda_2 = U_G$$

$$\frac{e_{SI}^2}{4\pi\epsilon_0\ r_{SI}}\ \lambda_1^2\,\lambda_2 = \frac{e_G^2}{\lambda_1\, r_{SI}}$$

$$e_G = \frac{\sqrt{\lambda_1^3\,\lambda_2}}{\sqrt{4\pi\epsilon_0}}\ e_{SI}$$

$$\rho_G = \frac{e_G}{r_G^3} = \frac{\sqrt{\lambda_1^3\,\lambda_2}}{\sqrt{4\pi\epsilon_0}}\ e_{SI}\ \frac{1}{\lambda_1^3\, r_{SI}}$$

$$\rho_G = \sqrt{\frac{\lambda_2}{\lambda_1^3}}\ \frac{1}{\sqrt{4\pi\epsilon_0}}\ \rho_{SI}$$

(4) In particular

$$e_G = \frac{\sqrt{\lambda_1^3\,\lambda_2}}{\sqrt{4\pi\epsilon_0}}\ e_{SI} = c_{SI}\sqrt{\frac{\mu_0}{4\pi}}\ \sqrt{\lambda_1^3\,\lambda_2}\ e_{SI}$$

$$= c_{SI} \times 10 \times e_{SI}$$

$$= 3\times 10^8 \times 10 \times \underbrace{1.6\times 10^{-19}\ C}_{\text{stat } C}$$

$$= 4.8 \times 10^{-10}\ \text{esu}.$$

(4) Let us next consider the electric field

$$E_G = ?\ \sqrt{4\pi\epsilon_0}\ E_{SI}.$$

We use the Lorentz force equation

$$F_G = e_G E_G \qquad [F] = MLT^{-2}$$

$$F_{SI} = e_{SI} E_{SI}$$

$$F_{SI}\,\lambda_1\lambda_2 = F_G$$

$$e_{SI} E_{SI}\,\lambda_1\lambda_2 = e_G E_G$$

$$e_{SI} E_{SI}\,\lambda_1\lambda_2 = \frac{\sqrt{\lambda_1^3\lambda_2}\; e_{SI}}{\sqrt{4\pi\epsilon_0}} E_G$$

$$E_G = \sqrt{\frac{\lambda_2}{\lambda_1}}\ \sqrt{4\pi\epsilon_0}\ E_{SI}$$

$$= \sqrt{\frac{\lambda_2}{\lambda_1}}\ \sqrt{\frac{4\pi}{\mu_0}}\ \frac{1}{c_{SI}}\ E_{SI}$$

$$= \frac{1}{10^{-4}}\ \frac{1}{c_{SI}}\ E_{SI}$$

$$\frac{\text{stat V}}{\text{cm}} = \frac{1}{3}\times 10^{4}\ \frac{\text{V}}{\text{m}}$$

(5) What about current ?

$$\frac{F_{SI}}{L_{SI}} = \frac{\mu_0}{2\pi} \frac{I_{SI}^2}{r_{SI}}$$

$$\frac{F_G}{L_G} = \frac{2}{c_{SI}^2} \frac{I_G^2}{r_G}$$

$$\frac{F_{SI}}{L_{SI}} \lambda_1 \lambda_2 = \frac{F_G}{L_G} = \frac{F_G}{\lambda_1 L_{SI}}$$

$$\frac{\mu_0}{2\pi} \frac{I_{SI}^2}{r_{SI}} \lambda_1^2 \lambda_2 = \frac{2}{c_{SI}^2} \frac{I_G^2}{\lambda_1 r_{SI}}$$

$$I_G^2 = C_{SI}^2 \lambda_1^3 \lambda_2 \frac{\mu_0}{4\pi} I_{SI}^2$$

$$I_G = \frac{\sqrt{\lambda_1^3 \lambda_2}}{\sqrt{4\pi \epsilon_0}} I_{SI}$$

$$= C_{SI} \times 10 \times I_{SI}$$

$$1 \text{ stat A} = 3 \times 10^9 \text{ A}$$

(6) Conservation of charge

$$\frac{\partial}{\partial t}\, \vec{\nabla}\cdot\vec{D} = \frac{\partial \rho}{\partial t}$$

$$\vec{\nabla}\cdot\frac{\partial \vec{D}}{\partial t} = \frac{\partial \rho}{\partial t}$$

$$\vec{\nabla}\cdot\left[\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \vec{J}\right] = \frac{\partial \rho}{\partial t}$$

$$\frac{1}{\mu_0}\,\underbrace{\vec{\nabla}\cdot\vec{\nabla}\times\vec{B}}_{=0} = \frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

(7)
$$\int_V d^3r\, \frac{\partial \rho}{\partial t} + \int_V d^3r\, \vec{\nabla}\cdot\vec{J} = 0$$

$$\frac{\partial}{\partial t}\int_V d^3r\, \rho + \oint_S d\vec{a}\cdot\vec{J} = 0$$

$$\frac{\partial}{\partial t} Q_{in} + \oint_S d\vec{a}\cdot\vec{J} = 0$$

$\vec{J} \to$ flux of charge.

rate of change of charge inside volume V

rate of flow of charge across surface S.

$\to$ conservation of charge is locally valid.