Midterm Exam No. 02 (Fall 2014)

PHYS 520A: Electromagnetic Theory I

Date: 2014 Oct 28

1. (**25 points.**) Evaluate
$$\boldsymbol{\nabla} \cdot \left(\frac{\mathbf{r}}{r^3} \right), \tag{1}$$
everywhere in space, including $\mathbf{r} = 0$.
Hint: Check your answer for consistency by using divergence theorem.

2. (**25 points.**) The response of a material to an electric field, in a particular model, is described by the susceptibility function
$$\chi(\omega) = \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma}, \tag{2}$$
where ω_p, ω_0, and γ are material dependent parameters, and ω is the frequency of oscillation of the electric field.

   (a) $[\mathrm{Re}\chi(\omega)]$ is a measure of the square of the refractive index. Plot $[\mathrm{Re}\chi(\omega)]$ as a function of ω.

   (b) $[\mathrm{Im}\chi(\omega)]$ is a measure of absorption of light. Plot $[\mathrm{Im}\chi(\omega)]$ as a function of ω.

3. (**25 points.**) A permanently polarized sphere of radius R is described by the polarization vector
$$\mathbf{P}(\mathbf{r}) = \alpha r^2 \, \hat{\mathbf{r}} \, \theta(R - r). \tag{3}$$
Find the effective charge density by calculating $-\boldsymbol{\nabla} \cdot \mathbf{P}$. In particular, you should obtain two terms, one containing $\theta(R - r)$ that is interpreted as a volume charge density, and another containing $\delta(R - r)$ that can be interpreted as a surface charge density.

4. (**25 points.**) A particle of mass m and charge q moving in a uniform magnetic field $\mathbf{B}$ experiences a velocity dependent force $\mathbf{F}$ given by the expression
$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}, \tag{4}$$
where $\mathbf{v}(t) = d\mathbf{x}/dt$ is the velocity of the particle in terms of its position $\mathbf{x}(t)$. Choose the magnetic field to be along the positive z direction, $\mathbf{B} = B\hat{\mathbf{z}}$.

   (a) Using initial conditions $\mathbf{v}(0) = 0\,\hat{\mathbf{x}} + v_0\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{z}}$ and $\mathbf{x}(0) = 0\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{z}}$, solve the differential equation in Eq. (4) to find the position $\mathbf{x}(t)$ and velocity $\mathbf{v}(t)$ as a function of time.

(b) In particular, prove that the particle takes a circular path. Determine the radius of this circular path and the position of the center of the circular path.

(c) What is the precession frequency ω_p of the particle (and thus that of the velocity vector $\mathbf{v}$) about the magnetic field $\mathbf{B}$? Is the precession frequency determined by the initial conditions to the differential equation?