Homework No. 02 (Fall 2014)

PHYS 520A: Electromagnetic Theory I

Due date: Tuesday, 2014 Sep 9, 4.00pm

1. The Lorentz force law in SI units is

$$\mathbf{F} = q\big[\mathbf{E} + \mathbf{v} \times \mathbf{B}\big]. \tag{1}$$

Write down the Lorentz force law in Lorentz-Heaviside units.

2. In Gaussian units the power radiated by an accelerated charged particle of charge e is given by the Larmor formula,

$$P = \frac{2\,e^2}{3\,c^3}\,a^2, \tag{2}$$

where a is the acceleration of the charged particle. Write down the Larmor formula in SI units, and in Lorentz-Heaviside units.

3. In Gaussian units the cyclotron frequency is

$$\omega_0 = \frac{eB}{mc}, \tag{3}$$

where m is the mass of electron. Write down the expression for cyclotron frequency in SI units, and in Lorentz-Heaviside units.

4. (Ref. Schwinger et al., problem 1, chapter 1.) For an arbitrarily moving charge, the charge and current densities are

$$\rho(\mathbf{r}, t) = q\delta(\mathbf{r} - \mathbf{r}_a(t)) \tag{4}$$

and

$$\mathbf{j}(\mathbf{r}, t) = q\mathbf{v}_a(t)\,\delta(\mathbf{r} - \mathbf{r}_a(t)), \tag{5}$$

where $\mathbf{r}_a(t)$ is the position vector and

$$\mathbf{v}_a(t) = \frac{d\mathbf{r}_a}{dt} \tag{6}$$

is the velocity of the charged particle. Verify the statement of conservation of charge,

$$\frac{\partial}{\partial t}\rho(\mathbf{r}, t) + \boldsymbol{\nabla} \cdot \mathbf{j}(\mathbf{r}, t) = 0. \tag{7}$$

5. (Ref. Schwinger et al., problem 7, chapter 1.) A charge e moves in the vacuum under the influence of uniform fields $\mathbf{E}$ and $\mathbf{B}$. Assume that $\mathbf{E} \cdot \mathbf{B} = 0$ and $\mathbf{v} \cdot \mathbf{B} = 0$. At what velocity does the charge move without acceleration? What is the speed when $|\mathbf{E}| = |\mathbf{B}|$?