Homework No. 04 (Fall 2014)

PHYS 520A: Electromagnetic Theory I

Due date: Thursday, 2014 Sep 25, 4.00pm

1. (Ref. Schwinger et al., problem 4.1.) Consider the charge density

$$\rho(\mathbf{r}) = -\mathbf{d} \cdot \boldsymbol{\nabla} \delta^{(3)}(\mathbf{r}). \tag{1}$$

   (a) Find the total charge of the charge density by evaluating

$$\int d^3r \, \rho(\mathbf{r}). \tag{2}$$

   (b) Find the dipole moment of the charge density by evaluating

$$\int d^3r \, \mathbf{r} \, \rho(\mathbf{r}). \tag{3}$$

2. The magnetic dipole moment of charge q_a moving with velocity $\mathbf{v}_a$ is

$$\boldsymbol{\mu} = \frac{1}{2} q_a \mathbf{r}_a \times \mathbf{v}_a, \tag{4}$$

   where $\mathbf{r}_a$ is the position of the charge. For a charge moving along a circular orbit of radius r_a, with constant speed v_a, deduce the magnetic moment

$$\boldsymbol{\mu} = I A \hat{\mathbf{n}}, \qquad I = \frac{q_a}{\Delta t} \frac{v_a \Delta t}{2\pi r_a} \qquad A = \pi r_a^2, \tag{5}$$

   where $\hat{\mathbf{n}}$ points along $\mathbf{r}_a \times \mathbf{v}_a$.

3. Identify the orbital angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ in the expression for magnetic dipole moment, then generalize to total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, where $\mathbf{S}$ is the spin of the particle. Thus, deduce the relation

$$\boldsymbol{\mu} = \gamma \mathbf{J}, \tag{6}$$

   where γ is the gyromagnetic ratio of a particle. A magnetic dipole moment feels a torque given by

$$\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt} = \boldsymbol{\mu} \times \mathbf{B}, \tag{7}$$

   which causes the magnetic moment to precess around the magnetic field. Solve the above equations and find the precession angular frequency in terms of γ and B.

4. Show that the effective charge density, ρ_{eff}, and the effective current density, $\mathbf{j}_{\text{eff}}$,

$$\rho_{\text{eff}} = -\boldsymbol{\nabla} \cdot \mathbf{P}, \tag{8}$$

$$\mathbf{j}_{\text{eff}} = \frac{\partial}{\partial t}\mathbf{P} + \boldsymbol{\nabla} \times \mathbf{M}, \tag{9}$$

satisfy the equation of charge conservation

$$\frac{\partial}{\partial t}\rho_{\text{eff}} + \boldsymbol{\nabla} \cdot \mathbf{j}_{\text{eff}} = 0. \tag{10}$$