Homework No. 05 (Fall 2014)

PHYS 520A: Electromagnetic Theory I

Due date: Thursday, 2014 Oct 16, 4.00pm

1. Consider a solid cylinder of radius R and infinite length with uniform permanent polarization

$$\mathbf{P}(\mathbf{r},t) = \mathbf{P}_0\,\theta(R-\rho), \tag{1}$$

where $\rho^2 = x^2 + y^2$ and $\mathbf{P}_0$ is perpendicular to the axis of the cylinder. We shall find the electric potential and the electric field outside the cylinder.

(a) Show that the effective charge density is given by the expression

$$\rho_{\text{eff}}(\mathbf{r}) = -\boldsymbol{\nabla}\cdot\mathbf{P} = \mathbf{P}_0\cdot\hat{\boldsymbol{\rho}}\,\delta(\rho-R). \tag{2}$$

(b) Beginning from

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho_{\text{eff}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}, \tag{3}$$

after integrating by parts, and writing

$$\phi(\mathbf{r}) = -\frac{1}{4\pi\varepsilon_0}\mathbf{P}_0\cdot\boldsymbol{\nabla}\int d^3r'\,\frac{\theta(R-\rho')}{|\mathbf{r}-\mathbf{r}'|}, \tag{4}$$

show that

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\theta(R-\rho')\,\frac{\mathbf{P}_0\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}. \tag{5}$$

(c) Evaluate the integrals, z', ϕ', and ρ', to show that the electric potential outside the cylinder is given by

$$\phi(\mathbf{r}) = \frac{2\pi R^2}{4\pi\varepsilon_0}\,\frac{\mathbf{P_0}\cdot\boldsymbol{\rho}}{\rho^2}. \tag{6}$$

(d) Evaluate the gradient of the electric potential to show that the electric field outside the cylinder is given by

$$\mathbf{E}(\mathbf{r}) = \frac{2\pi R^2}{4\pi\varepsilon_0}\,\frac{1}{\rho^2}\Big[2(\mathbf{P_0}\cdot\hat{\boldsymbol{\rho}})\hat{\boldsymbol{\rho}} - \mathbf{P}_0\Big]. \tag{7}$$