Solutions to Midterm Exam 02 (Spring 2015)

prob 1, MT-02

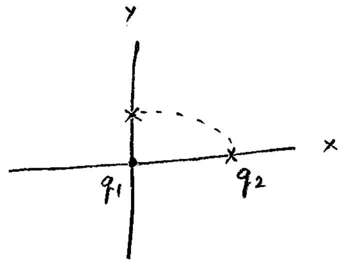
$$\frac{1}{2} m v^2 = q V$$

$$v = \sqrt{\frac{2 q V}{m}} = \sqrt{\frac{2 \times 1.6\times10^{-19} \times 10}{9.1\times 10^{-31}}} = 1.88 \times 10^{6} \ \frac{m}{s}$$

prob. 2, MT-02

$q_1 = 7.2\times10^{-8}C$.

$q_2 = -2.7\times10^{-8}C$



$$U_i = \frac{k q_1 q_2}{r_i} = -\frac{9\times10^{9} \times 7.2\times10^{-8} \times 2.7\times10^{-8}}{3.5\times10^{-2}}$$

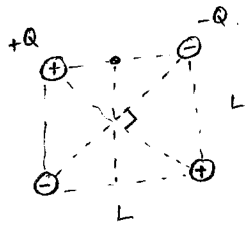
$$= -5.0 \times 10^{-4} \text{ J}$$

$$U_f = \frac{k q_1 q_2}{r_f} = -\frac{9\times10^{9} \times 7.2\times10^{-8} \times 2.7\times10^{-8}}{2.5\times10^{-2}}$$

$$= -7.0 \times 10^{-4} \text{ J}$$

$$U_f - U_i = (-7.0 + 5.0)\times10^{-4} \text{ J} = -2.0\times10^{-4} \text{ J}$$

prob. 3, MT-02



(a) $$V_1 = \frac{kQ}{x} - \frac{kQ}{x} + \frac{kQ}{x} - \frac{kQ}{x} = 0$$

(b) $$V_2 = \frac{kQ}{L/2} - \frac{kQ}{L/2} + \frac{kQ}{\sqrt{(\frac{L}{2})^2+L^2}} - \frac{kQ}{\sqrt{(\frac{L}{2})^2+L^2}} = 0$$

(c) $V_1 - V_\infty = 0.$

(d) $V_2 - V_1 = 0.$

prob. 4, MT-02

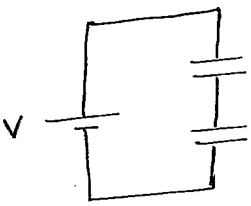
Given $\frac{R_2}{R_1} = 2$

$V_1 = V_2$

$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{R_2}{R_1} = 2$

$$\frac{E_2}{E_1} = \frac{\left(\frac{kQ_2}{R_2^2}\right)}{\left(\frac{kQ_1}{R_1^2}\right)} = \frac{Q_2}{Q_1}\left(\frac{R_1}{R_2}\right)^2 = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}.$$

prob. 5, MT-02



(a) $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$

$C_{eq} = \frac{20}{3}\mu F = 6.67\mu F$

(b) $Q_{eq} = C_{eq} V = \frac{20}{3}\mu F \; 10V = \frac{200}{3}\mu C.$

$Q_1 = Q_2 = Q_{eq} = \frac{200}{3}\mu C = 66.7\mu C.$

(c) $V_1 = \frac{Q_1}{C_1} = \frac{200}{3}\frac{\mu C}{10\mu F} = \frac{20}{3} V = 6.7 V$

$V_2 = \frac{Q_2}{C_2} = \frac{200}{3}\frac{\mu C}{20\mu F} = \frac{10}{3} V = 3.3 V$

(d) $U_1 = \frac{Q_1^2}{2C_1} = \frac{1}{2}\left(\frac{200}{3}\mu C\right)^2 \frac{1}{10\mu F} = 222.2\mu J$

$U_2 = \frac{Q_2^2}{2C_2} = \frac{1}{2}\left(\frac{200}{3}\mu C\right)^2 \frac{1}{20\mu F} = 111.1\mu J$

prob. 6, MT-02

$\vec{\tau} = \vec{p} \times \vec{E}$

$= 3.0\times 10^{-12} \times 16 \text{ N-m } \hat{i}\times\hat{j}.$

$= 48\times 10^{-12} \text{ N-m } \hat{k}$

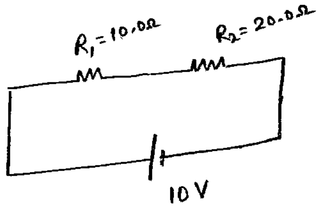
$= 4.8\times 10^{-11} \text{ N-m } \hat{k}$

prob. 7, MT-02

$$1 \text{ kWh} = 10^3 \frac{J}{s}\ 60 \times 60\ s = 36 \times 10^5\ J$$

$$\text{Energy used} = \frac{50.00 \text{ USD}}{0.10 \text{ USD}}\ 36 \times 10^5\ J = 1.8 \times 10^9 \text{ Joules}.$$

prob 8, MT-02



(a) $R_{eq} = R_1 + R_2 = 30.0\,\Omega$

(b) $I_{eq} = \frac{V}{R_{eq}} = \frac{10}{30.0} = 0.33$ A

$I_1 = I_2 = I_{eq} = 0.33$ A

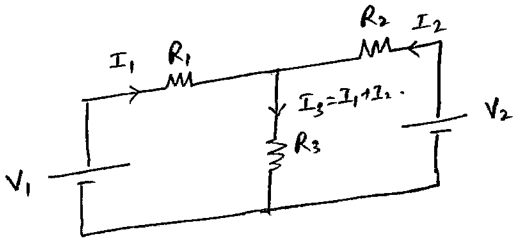
(c) $V_1 = I_1 R_1 = \frac{1}{3} \times 10.0 = \frac{10}{3}$ V $= 3.33$ V

$V_2 = I_2 R_2 = \frac{1}{3} \times 20.0 = \frac{20}{3}$ V $= 6.67$ V

(d) $P_1 = I_1^2 R_1 = \left(\frac{1}{3} A\right)^2 (10.0\,\Omega) = 1.11$ W

$P_2 = I_2^2 R_2 = \left(\frac{1}{3} A\right)^2 (20.0\,\Omega) = 2.22$ W

prob. 9, MT-02



$$V_1 - I_1 R_1 - (I_1 + I_2) R_3 = 0$$

$$V_2 - I_2 R_2 - (I_1 + I_2) R_3 = 0$$

Thus,

$$\left.\begin{aligned} 40.0\, I_1 + 30.0\, I_2 &= 10 \\ 30.0\, I_1 + 50.0\, I_2 &= 20 \end{aligned}\right\} \Rightarrow$$

$I_1 = \frac{500 - 600}{2000 - 900} = -\frac{1}{11}$ A

$I_2 = \frac{800 - 300}{2000 - 900} = +\frac{5}{11}$ A

$I_3 = I_1 + I_2 = \frac{4}{11}$ A $= 0.36$ A

prob. 10, MT-02

$$Q(t) = Q\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$\frac{2}{3} Q = Q\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$e^{-\frac{t}{\tau}} = 1 - \frac{2}{3} \Rightarrow t = \tau \ln 3.$$