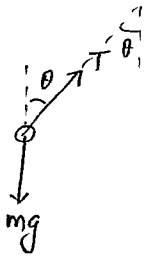
(Solutions to MT03, Fall 2015, PHYS 205A)

Solutions

Prob. 1



$$\vec{T} + m\vec{g} = m\vec{a}$$

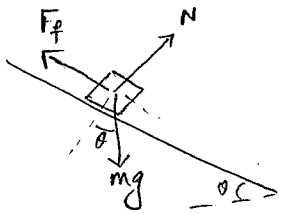
x: $T\sin\theta + 0 = ma \Rightarrow T\sin\theta = ma$

y: $T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg$

(a) $\theta = \tan^{-1}\left(\frac{a}{g}\right)$

(b) $T = m\sqrt{a^2+g^2}$

Prob 2



(a) $W_N = \vec{N}\cdot\vec{d} = N d\cos 90 = 0$

(b) $W_g = m\vec{g}\cdot\vec{d} = mgd\cos(90-\theta)$
$= mgd\sin\theta$
$= 20.0\times 9.8\times 10.0\sin 30$
$= 980\ \text{J}$

$F_f = \mu_k N$
$= \mu_k mg\cos\theta$

(c) $W_f = \vec{F}_f\cdot\vec{d} = F_f d\cos 180$
$= -\mu_k mg\cos\theta\, d$
$= -0.30\times 20.0\times 9.8\times\cos 30\times 10.0$
$= -509\ \text{J}$

(d) $\Delta K = W_N + W_g + W_f$
$= 0 + 980 - 509 = 471\ \text{J}$

Prob 3

$U_A + K_A = U_B + K_A$

$mgh_A + \frac{1}{2}mv_A^2 = mgh_B + \frac{1}{2}mv_B^2$ ($v_A = 0$)

$\sqrt{2g(h_A - h_B)} = v_B$

$v_B = \sqrt{2\times 9.8\times(7.00-3.20)} = 8.63\ \frac{\text{m}}{\text{s}}$

Prob. 4

$U_A^g = mgh_A$

$U_A^s = \frac{1}{2} k x_A^2$

$K_A = \frac{1}{2} m v_A^2$

(a) $U_A^g + U_A^s + K_A = U_B^g + U_B^s + K_B$

$mgh + 0 + 0 = 0 + 0 + \frac{1}{2} m v_B^2$

$v_B = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.0} = 4.43 \frac{m}{s}$

(b) $U_B^g + U_B^s + K_B = U_C^g + U_C^s + K_C$

$0 + 0 + \frac{1}{2} m v_B^2 = 0 + \frac{1}{2} k x_c^2 + 0$

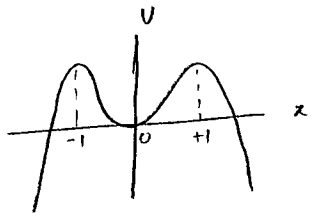
$\frac{1}{2} m v_B^2 = \frac{1}{2} k x_c^2$

$x_c = \sqrt{\frac{m}{k}}\, v_B = \sqrt{\frac{20.0\ kg}{2.0 \times 10^4 \frac{N}{m}}} \times 4.43 \frac{m}{s} = 0.14\ m$

Prob. 5

(a) $U(x) = a x^2 - b x^4$

$= \frac{1}{2} x^2 - \frac{1}{4} x^4$



(b) $F = -\frac{dU}{dx}$

$= -x + x^3$

$= x(x^2 - 1)$

$F = 0 \Rightarrow x(x^2-1) = 0 \Rightarrow x = 0, -1, +1$

Prob. 6

$m_B v_{Bi} + m_\omega \underbrace{v_{\omega i}}_{=0} = (m_B + m_\omega) v_f$

$v_{Bi} = \left(1 + \frac{m_\omega}{m_B}\right) v_f$

$= \left(1 + \frac{5000}{10}\right) 0.600 \frac{m}{s}$

$= 300.6 \frac{m}{s}$

Prob. 7

$$m_e v_{ei} + m_H v_{Hi} = m_e v_{ef} + m_H v_{Hf}$$

$$\frac{1}{2} m_e v_{ei}^2 + \frac{1}{2} m_H v_{Hi}^2 = \frac{1}{2} m_e v_{ef}^2 + \frac{1}{2} m_H v_{Hf}^2$$

$$v_{ef} = \left(\frac{m_e - m_H}{m_e + m_H}\right) v_{ei} + \left(\frac{2 m_H}{m_e + m_H}\right) \underbrace{v_{Hi}}_{=0}$$

$$v_{Hf} = \left(\frac{2 m_e}{m_e + m_H}\right) v_{ei} + \left(\frac{m_H - m_e}{m_e + m_H}\right) v_{Hi}$$

$$\frac{K_{Hf}}{K_{ei}} = \frac{\frac{1}{2} m_H v_{Hf}^2}{\frac{1}{2} m_e v_{ei}^2} = \frac{m_H}{m_e}\left(\frac{2 m_e}{m_e + m_H}\right)^2 = 1837\left(\frac{2}{1+1837}\right)^2 = \frac{1}{460}$$

$$= 0.00217$$

Prob. 8

$$\frac{dm}{dx} = a x = \lambda$$

$$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^L x\, a x\, dx}{\int_0^L a x\, dx}$$

$$= \frac{a \int_0^L x^2 dx}{a \int_0^L x\, dx}$$

$$= \frac{\frac{L^3}{3}}{\frac{L^2}{2}} = \frac{2}{3} L = 0.67\ m$$