Solutions to Final Exam, PHYS 205A, Fall 2015

Prob. 1

$x(t) = 54t^2 - 3.0t^4$

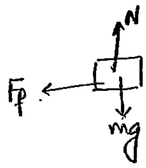
$v(t) = \frac{dx}{dt} = 108t - 12t^3$

$a(t) = \frac{d^2x}{dt^2} = 108 - 36t^2$

Stops at $v(t) = 0 \Rightarrow t = 0,\ t = \pm 3$. Thus $t = +3.0$ seconds.

$a(3.0) = 108 - 36(3.0)^2 = -216\ \frac{m}{s^2}$

Prob. 2

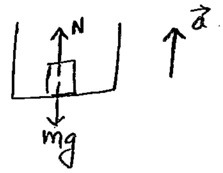


$-\mu_k N = ma.$

$-\mu_k mg = ma \Rightarrow a = -\mu_k g$

$\Delta x = \frac{V_f^2 - V_i^2}{2a} = \frac{0 - V_i^2}{-2\mu_k g} = \frac{V_i^2}{2\mu_k g} = \frac{25.0^2}{2 \times 0.50 \times 9.8} = 64\ m.$

Prob. 3



Acceleration is upward.

$a = \frac{V_f - V_i}{\Delta t} = \frac{0 - 15}{3.0} = -5.0\ m/s^2.$

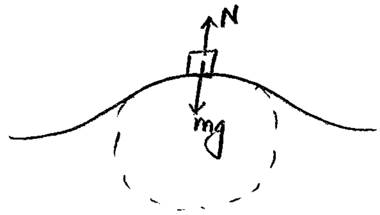
$|\vec{a}| = 5.0\ m/s^2$

$N - mg = ma$

$N = m(g + a) = 85\ kg\ (9.8 + 5.0)\ m/s^2$

$= 1258\ N$

Prob. 4



$$mg - N = \frac{mv^2}{R}$$

At greatest speed, $N = 0$.

$$\Rightarrow \quad v = \sqrt{gR}$$

$$= \sqrt{9.8 \times 250} = 49.5 \text{ m/s}$$

Prob. 5

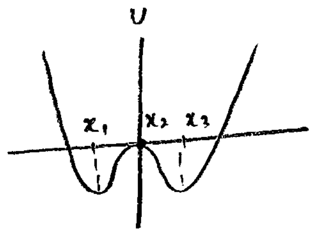
$$mgh_A + \underbrace{\tfrac{1}{2} m v_A^2}_{=0} + \underbrace{\tfrac{1}{2} k x_A^2}_{=0} = \underbrace{mgh_B}_{=0} + \underbrace{\tfrac{1}{2} m v_B^2}_{=0} + \tfrac{1}{2} k x_B^2$$

$$mgh_A = \frac{1}{2} k x_B^2.$$

$$x_B = \sqrt{\frac{2mgh_A}{k}} = \sqrt{\frac{2 \times 20.0 \times 9.8 \times 1.0}{2.0 \times 10^4}} = 0.14 \text{ m} = 14 \text{ cm}$$

Prob. 6

(a)



$$x_1 = -\sqrt{\frac{a}{2b}} = -1 \text{ m}$$

$$x_2 = 0$$

$$x_3 = +\sqrt{\frac{a}{2b}} = +1 \text{ m}$$

(b)

$$F = -\frac{\partial U}{\partial x} = 2ax - 4bx^3$$

$$F = 0 \quad \Rightarrow \quad x = 0 \quad \text{and} \quad x = \pm\sqrt{\frac{a}{2b}}$$

$$\Rightarrow \quad x = 0 \quad \text{and} \quad x = \pm 1 \text{ m}$$

$x = 0$: unstable equilibrium

$x = \pm 1$m : stable equilibrium.

Prob. 7

B - bullet
W - Wood.

$$m_B v_i = (m_B + M_W) v_f$$

$$v_f = \frac{m_B}{(m_B + M_W)} v_i = \frac{10.0}{(10.0 + 5000)} \times 2500 \frac{km}{h}$$

$$= 4.99 \frac{km}{hour} = 1.39 \frac{m}{s}$$

Prob. 8

$I = \frac{2}{5} M R^2$

$\omega_i = \frac{v_i}{R}$

$\omega_f = \frac{v_f}{R}$

$$M g h_i + \frac{1}{2} M v_i^2 + \frac{1}{2} I \omega_i^2 = M g h_f^{=0} + \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2$$

$$M g h + \frac{1}{2} M v_i^2 + \frac{1}{2} \frac{2}{5} M v_i^2 = 0 + \frac{1}{2} M v_f^2 + \frac{1}{2} \frac{2}{5} M v_f^2$$

$$\Rightarrow \quad g h + \frac{7}{10} v_i^2 = \frac{7}{10} v_f^2$$

$$v_f = \sqrt{v_i^2 + \frac{10}{7} g h} = \sqrt{4.00^2 + \frac{10}{7} \times 9.8 \times 0.25} = 4.42 \ m/s$$

Prob. 9

$\Delta t = ?$

$\Delta\theta = ?$

$$\omega_i = 1.00 \times 10^2 \frac{rev}{min} \frac{2\pi \ rad}{1 \ rev} = 10.5 \frac{rad}{s}$$

$$\omega_f = 0$$

$$\alpha = -2.00 \frac{rad}{s^2}$$

(a) $$\Delta t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 10.5 \frac{rad}{s^2}}{-2.00 \frac{rad}{s^2}} = 5.25 \text{ seconds.}$$

(b) $$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0 - 10.5^2}{2 \times (-2.00)} = 27.6 \ rad = 4.39 \text{ revolutions}$$

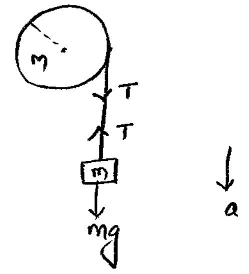
Prob. 10

m: $mg - T = ma$

M: $TR = I\alpha$

$\alpha R = a$

$I = \frac{1}{2}MR^2$



$$mg = ma + \frac{I}{R}\alpha$$

$$= ma + \frac{I}{R^2}a$$

$$mg = \left(m + \frac{1}{2}M\right)a$$

$$a = \frac{m}{\left(m + \frac{1}{2}M\right)} g = \frac{5.0}{\left(5.0 + \frac{3.00}{2}\right)} 9.8 = 7.54 \frac{m}{s^2}$$

Prob. 11

$I_i = I_s + 2m r_i^2$

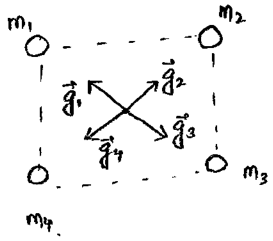
$I_f = I_s + 2m r_f^2$

$$I_i \omega_i = I_f \omega_f$$

$$\omega_f = \frac{\left(I_s + 2m r_i^2\right)}{\left(I_s + 2m r_f^2\right)} \omega_i$$

$$= \frac{\left(2.60 + 2 \times 3.00 \times 1.00^2\right)}{\left(2.60 + 2 \times 3.00 \times 0.30^2\right)} 0.75 \frac{rad}{s} = 2.05 \frac{rad}{sec}$$

Prob. 12



$$|\vec{g}_1| = |\vec{g}_2| = |\vec{g}_3| = |\vec{g}_4| = \frac{Gm}{2L^2}$$

$\vec{g}_1$ cancels $\vec{g}_3$ and $\vec{g}_2$ cancels $\vec{g}_4$.

Thus, $\vec{g}_{tot} = 0$.