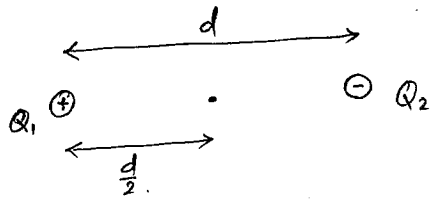
Solution (MT-02, PHYS-205B, 2015 Fall)

Prob. 1

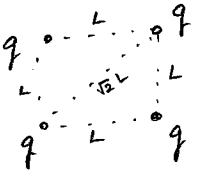


$$V = \frac{kQ_1}{d/2} + \frac{kQ_2}{d/2}$$

$$= + \frac{9.0\times10^{9} \times 4.0\times10^{-9}}{(1\,m/2)} - \frac{9.0\times10^{9} \times 2.0\times10^{-9}}{(1\,m/2)}$$

$$= +72 - 36 = 36 \text{ Volt}$$

Prob. 2



$$U_i = \frac{kq^2}{L} + \frac{kq^2}{L} + \frac{kq^2}{\sqrt{2}\,L} = \left(2+\frac{1}{\sqrt{2}}\right)\frac{kq^2}{L}$$

$$U_f = U_i + \frac{kq^2}{L} + \frac{kq^2}{L} + \frac{kq^2}{\sqrt{2}\,L} = 2U_i$$

$$U_f - U_i = U_i = \left(2+\frac{1}{\sqrt{2}}\right)\frac{kq^2}{L}$$

$$= \left(2+\frac{1}{\sqrt{2}}\right)\frac{8.99\times10^{9} \times (1.00\times10^{-6})^2}{(0.100\,m)} = 0.243 \text{ Joules.}$$

Prob. 3

$$E_x = -\frac{\partial}{\partial x}V = -b$$

$$E_y = -\frac{\partial}{\partial y}V = 0$$

$$E_z = -\frac{\partial}{\partial z}V = 0$$

$$\vec{E} = -b\,\hat{i}$$

$$= +450\,\hat{i}\ \frac{V}{cm}$$

$$= \hat{i}\ 4.5\times10^{4}\ \frac{V}{m}.$$

Prob. 4

Electric field is zero inside the conductor.

Electric potential is uniform inside a conductor.

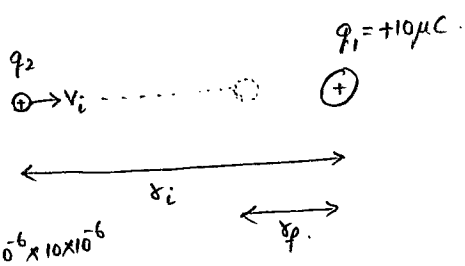
$$E = 0$$

$$V = \frac{kQ}{R} = \frac{9.0\times10^{9} \times 1.0\times10^{-6}}{0.10\,m} = 9.0\times10^{4}\ V$$

Prob. 5

$$\frac{1}{2} m_2 v_{2i}^2 + \frac{k q_1 q_2}{r_i} = \frac{1}{2} m_2 v_{2f}^2 + \frac{k q_1 q_2}{r_f}$$

($v_{2f} = 0$)



$$\frac{1}{2} 10^{-2} 10^2 + \frac{9.0\times10^9 \times 1.0\times10^{-6} \times 10\times10^{-6}}{0.30} = \frac{9.0\times10^9 \times 1.0\times10^{-6} \times 10\times10^{-6}}{r_f}$$

$$0.5 + 0.30 = \frac{0.090}{r_f} \Rightarrow r_f = 0.11\,m = 11\,cm.$$

Prob. 6

(a) $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20} \Rightarrow C_{eq} = 6.7\,\mu F$

(b) $Q_1 = Q_2 = C_{eq} V = 6.7\,\mu F \times 10\,V = 67\,\mu C$

(c) $V_1 = \frac{Q_1}{C_1} = \frac{67\,\mu C}{10\,\mu F} = 6.7\,V$

$V_2 = \frac{Q_2}{C_2} = \frac{67\,\mu C}{20\,\mu F} = 3.3\,V$

(d) $U_1 = \frac{Q_1^2}{2C_1} = \frac{(67\,\mu C)^2}{2\times 10\mu F} = 224\,\mu J$

$U_2 = \frac{Q_2^2}{2C_2} = \frac{(67\,\mu C)^2}{2\times 20\mu F} = 112\,\mu J$

Prob. 7

$V = Al$

$$R = \frac{\rho l}{A} = \frac{\rho}{V} l^2$$

$$R' = \frac{\rho}{V} l'^2$$

$$\frac{R'}{R} = \frac{l'^2}{l^2} = 100$$

$$R' = 100\,R = 1.000\times10^4\,\Omega$$

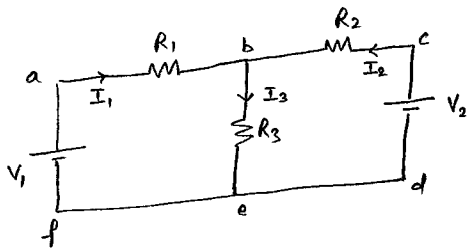
Prob. 8

$$R_{eq} = \frac{1.00\ k\Omega}{10} = 100.\ \Omega$$

$$I_{eq} = \frac{V}{R_{eq}} = \frac{10.0}{100} = 0.10\ A$$

$$I = \frac{I_{eq}}{10} = 0.010\ A$$

Prob. 9



junction b: $I_3 = I_1 + I_2$

loop fabef: $V_1 - I_1 R_1 - I_3 R_3 = 0$

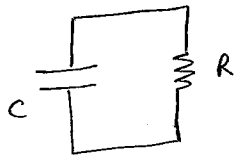
loop dcbed: $V_2 - I_2 R_2 - I_3 R_3 = 0$

$$(R_1 + R_3) I_1 + R_3 I_2 = V_1$$
$$R_3 I_1 + (R_2 + R_3) I_2 = V_2$$

$$\Rightarrow \quad 200\, I_1 + 100\, I_2 = 10$$
$$100\, I_1 + 200\, I_2 = 20$$

$$\Rightarrow \quad I_1 = 0$$
$$I_2 = 0.10\ A$$
$$I_3 = 0.10\ A$$

Prob. 10



$$Q(t) = Q_0\, e^{-\frac{t}{RC}}$$

$$\frac{1}{2} Q_0 = Q_0\, e^{-\frac{t}{RC}}$$

$$\ln \tfrac{1}{2} = -\frac{t}{RC}$$

$$t = RC \ln 2$$
$$= 1.40 \times 10^{-3} \ln 2$$
$$= 0.693 \times 1.40 \times 10^{-3}\ s$$
$$= 9.70 \times 10^{-4}\ s$$