

Preview of Midterm Exam No. 01 (Spring 2016)

PHYS 530A: Quantum Mechanics II

Date: 2016 Feb 19

1. **(20 points.)** The Hamiltonian for the motion of a particle of mass m in a constant gravitational field $\mathbf{g} = -g\hat{\mathbf{z}}$ is

$$H(z, p, t) = \frac{p^2}{2m} + mgz. \quad (1)$$

- (a) Show that the Hamilton equations of motion are

$$\frac{dz}{dt} = \frac{p}{m}, \quad (2a)$$

$$\frac{dp}{dt} = -mg. \quad (2b)$$

- (b) Show that the Hamilton-Jacobi equation

$$-\frac{\partial W}{\partial t} = H\left(z, \frac{\partial W}{\partial t}, t\right), \quad (3)$$

in terms of Hamilton's principal function $W(z, t)$ is given by

$$-\frac{\partial W}{\partial t} = \frac{1}{2m} \left(\frac{\partial W}{\partial t}\right)^2 + mgz. \quad (4)$$

Further, show that

$$W(z, t) = -Et - \frac{2\sqrt{2m}}{3mg}(E - mgz)^{\frac{3}{2}} \quad (5)$$

is a solution to the Hamilton-Jacobi equation up to a constant.

- (c) Hamilton's principal function allows us to identify canonical transformations $Q = Q(z, p, t)$ and $P = P(z, p, t)$, such that

$$\frac{\partial W}{\partial q} = p, \quad \frac{\partial W}{\partial Q} = -P, \quad \frac{\partial W}{\partial t} = -H, \quad (6a)$$

$$\frac{\partial W}{\partial p} = 0, \quad \frac{\partial W}{\partial P} = 0, \quad (6b)$$

with the feature that the new coordinates are constants of motion,

$$\frac{dQ}{dt} = 0 \quad \text{and} \quad \frac{dP}{dt} = 0. \quad (7)$$

To this end, choose $Q = E$ and then evaluate

$$P = -\frac{\partial W}{\partial Q} = t + \frac{p}{mg}. \quad (8)$$

Hint: Use $p = \frac{\partial W}{\partial q}$.

(d) Show that

$$Q = \frac{p^2}{2m} + mgz, \quad (9a)$$

$$P = t + \frac{p}{mg}, \quad (9b)$$

is a canonical transformation. That is, show that $[Q, P]_{q,p}^{\text{P.B.}} = 1$. Further, verify that

$$\frac{dQ}{dt} = 0, \quad (10a)$$

$$\frac{dP}{dt} = 0, \quad (10b)$$

$$K(Q, P, t) = H(z, p, t) + \frac{\partial W}{\partial t} = 0. \quad (10c)$$

2. **(20 points.)** Not available in preview mode.
3. **(20 points.)** Not available in preview mode.
4. **(20 points.)** Not available in preview mode.
5. **(20 points.)** Not available in preview mode.