Homework No. 03 (Spring 2016)

PHYS 530A: Quantum Mechanics II

Due date: Thursday, 2016 Feb 18, 4.30pm

1. (**20 points.**) (Ref: Milton's notes.) Using the notation for the probability for a measurement in the Stern-Gerlach experiment, introduced in the class, show that

$$p\big([+;\theta_1,\phi_1]\to[-;\theta_2,\phi_2]\big)=\frac{1-\cos\Theta}{2}, \tag{1}$$

where

$$\cos\Theta=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2). \tag{2}$$

2. (**20 points.**) (Ref: Milton's notes.) Show that

$$p\big([+;0,0]\to[+;\pi,0]\big)=0. \tag{3}$$

Further, show that

$$p\big([+;0,0]\to[\pm;\theta,\phi]\to[+;\pi,0]\big)=0, \tag{4}$$

which is a statement of destructive interference. Compare this with the probability for

$$p\big([+;0,0]\to[+;\theta,\phi]\to[+;\pi,0]\big) \tag{5}$$

and

$$p\big([+;0,0]\to[-;\theta,\phi]\to[+;\pi,0]\big). \tag{6}$$

3. (**20 points.**) Show that

$$p\big([+;0,0]\to[-;\pi,0]\big)=1. \tag{7}$$

Further, show that

$$p\big([+;0,0]\to[\pm;\theta,\phi]\to[-;\pi,0]\big)=1, \tag{8}$$

which is a statement of constructive interference. Compare this with the probability for

$$p\big([+;0,0]\to[+;\theta,\phi]\to[-;\pi,0]\big) \tag{9}$$

and

$$p\big([+;0,0]\to[-;\theta,\phi]\to[-;\pi,0]\big). \tag{10}$$

4. (**20 points.**) Using the properties of Pauli matrices,

$$\sigma_i\sigma_j=\delta_{ij}+i\varepsilon_{ijk}\sigma_k, \tag{11}$$

and the Euler formula

$$e^{ix}=\cos x+i\sin x, \tag{12}$$

evaluate

$$e^{-i\theta\frac{\sigma_x}{2}}\sigma_y e^{i\theta\frac{\sigma_x}{2}}. \tag{13}$$

What is the physical interpretation of this operation?