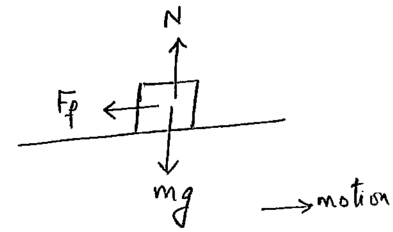
PHYS 205A, Fall 2016, Midterm Exam 2 Solutions

Prob. 1



$$m\vec{g} + \vec{N} + \vec{F}_f = m\vec{a}$$

x: $-F_f = ma$ y: $N - mg = 0$

$N = mg$.

$-\mu_s N = ma$

$-\mu_s \cancel{m}g = \cancel{m}a$

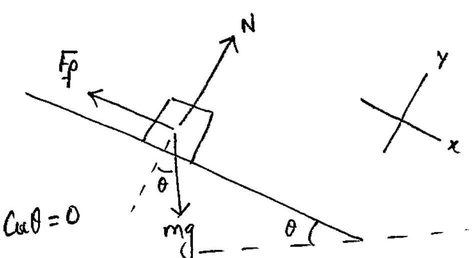
$a = -\mu_s g = -0.15 \times 9.8 = -1.47 \frac{m}{s^2}$

$V_i = +30.0 \frac{m}{s}$ $a = -1.47 \frac{m}{s}$ $\Delta x = ?$

$V_f = 0$ $\Delta t =$

$$\Delta x = \frac{V_f^2 - V_i^2}{2a} = \frac{0 - (30.0)^2}{2(-1.47)} = 306 \text{ m}$$

Prob. 2



$$m\vec{g} + \vec{N} + \vec{F}_f = m\vec{a}$$

x: $mg\sin\theta - F_f = ma_x$ y: $N - mg\cos\theta = 0$

$N = mg\cos\theta$

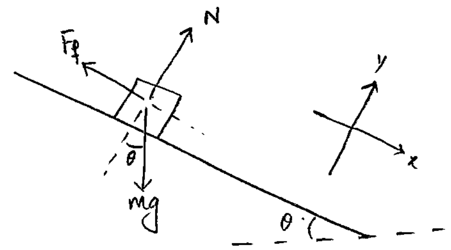
$mg\sin\theta = F_f \leq \mu_s N$

$\cancel{mg}\sin\theta \leq \mu_s \cancel{mg}\cos\theta$

$\tan\theta \leq \mu_s$ → condition for sliding to begin.

Since this condition does not change as water collects in the bucket, the bucket will never start sliding.

Answer: (d)



Prob. 3

$$m\vec{g} + \vec{N} + \vec{F_f} = m\vec{a}$$

x: $mg \sin\theta - F_f = ma$

y: $N - mg\cos\theta = 0$

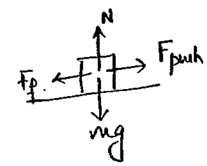
$N = mg\cos\theta$

$mg\sin\theta - \mu_k N = ma$

$\cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a$

$9.8 \sin 25 - \mu_k\, 9.8 \cos 25 = 2.5$

$\mu_k = 0.19$



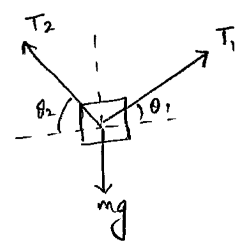
Prob. 4

Case 1 (static) $\vec{F}_{push} + \vec{F_f} + m\vec{g} + \vec{N} = 0$

$\Rightarrow$ $F_{push} - F_f = 0$

$F_{push} - \mu_s N = 0$

$F_{push} - \mu_s mg = 0$ $\Rightarrow$ $\mu_s = \dfrac{F_{push}}{mg} = \dfrac{80.0}{20.0 \times 9.8} = 0.408$



Prob. 5

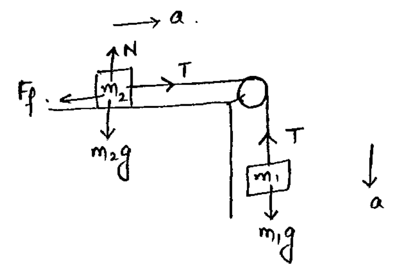
$$\vec{T_1} + \vec{T_2} + m\vec{g} = m\cancel{\vec{a}}^{=0}$$

x: $T_1\cos\theta_1 - T_2\cos\theta_2 = 0$

y: $T_1\sin\theta_1 + T_2\sin\theta_2 - mg = 0$

$$T_1 = \frac{mg\cos\theta_2}{\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2} = \frac{mg\cos\theta_2}{\sin(\theta_1+\theta_2)} = \frac{450\cos 60}{\sin(30+60)} = 225\ \text{N}$$

$$T_2 = \frac{mg\cos\theta_1}{\sin(\theta_1+\theta_2)} = \frac{450\cos 30}{\sin 90} = 390\ \text{N}$$



Prob. 6

m_1:

$$m_1\vec{g} + \vec{T} = m_1\vec{a}$$

$$m_1 g - T = m_1 a$$

m_2:

$$m_2\vec{g} + \vec{N} + \vec{T} + \vec{F_f} = m_2\vec{a}$$

x: $T - F_f = m_2 a$

y: $N - m_2 g = 0$

$N = m_2 g$

Initially at rest, so let $a = 0$

$$m_1 g = T$$

$$T = F_f \leq \mu_s N = \mu_s m_2 g$$

which together implies.

$$m_1 g \leq \mu_s m_2 g$$

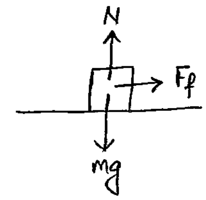
$$\frac{m_1}{m_2} \leq \mu_s \longrightarrow \text{condition for it not to move.}$$

$$\frac{1.0}{2.0} \leq 0.60$$

$$0.5 \leq 0.6$$

Since this condition is satisfied the masses will not start moving.

Prob. 7



$\longrightarrow v$ (direction of motion)

$\longrightarrow a$

(a) The tendency of cup is to move backward, due to inertia. So, the direction of friction is forward, in the direction of acceleration.

(b) $$m\vec{g} + \vec{N} + \vec{F_f} = m\vec{a}$$

x: $$F_f = ma$$

y: $$N - mg = 0$$
$$N = mg$$

$$ma = F_f \leq \mu_s N$$

$$\cancel{m} a \leq \mu_s \cancel{m} g$$

$$a \leq \mu_s g = 0.30 \times 9.8 = 2.94 \text{ m/s}^2$$

maximum acceleration

Prob. 8

$$v = v_T \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$0.75 \cancel{v_T} = \cancel{v_T} \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$0.75 = 1 - e^{-\frac{t}{\tau}}$$

$$e^{-\frac{t}{\tau}} = 1 - 0.75$$

$$e^{-\frac{t}{\tau}} = 0.25$$

$$\ln e^{-\frac{t}{\tau}} = \ln(0.25)$$

$$-\frac{t}{\tau} = \ln(0.25)$$

$$t = -\tau \ln(0.25)$$

$$= 1.39\,\tau.$$