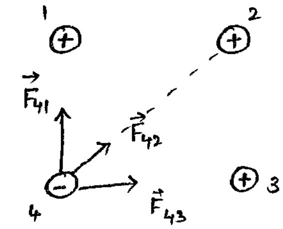
Solution — PHYS-205B, Midterm Exam 1, 2016 Fall

Prob. 1

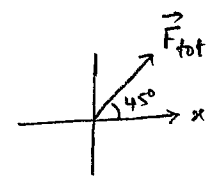


$$\vec{F}_{41} = 0\,\hat{i} + \frac{kQ^2}{L^2}\hat{j}$$

$$\vec{F}_{42} = \frac{kQ^2}{(\sqrt{2}L)^2}\cos 45\,\hat{i} + \frac{kQ^2}{(\sqrt{2}L)^2}\sin 45\,\hat{j}$$

$$\vec{F}_{43} = \frac{kQ^2}{L^2}\hat{i} + 0\,\hat{j}$$

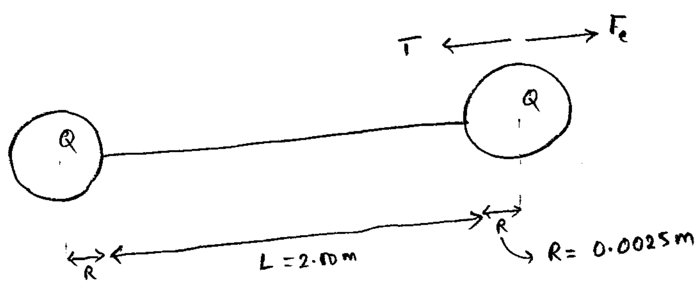
$$\vec{F}_{tot} = \frac{kQ^2}{L^2}\left(1+\frac{1}{2\sqrt{2}}\right)\hat{i} + \frac{kQ^2}{L^2}\left(1+\frac{1}{2\sqrt{2}}\right)\hat{j}$$



magnitude : $|\vec{F}_{tot}| = \frac{kQ^2}{L^2}\sqrt{2}\left(1+\frac{1}{2\sqrt{2}}\right) = \frac{kQ^2}{L^2}\left(\sqrt{2}+\frac{1}{2}\right)$

direction : 45° counterclockwise w.r.t. $+\hat{x}$.

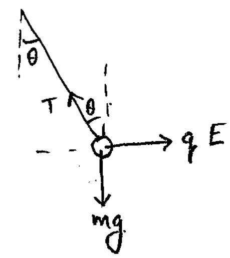
Prob. 2



$$Q = \frac{5.00\,\mu C}{2} = 2.50\times10^{-6}\,C$$

$$T = F_e = \frac{kQ^2}{(L+R+R)^2} = \frac{8.99\times10^{9}\,(2.50\times10^{-6})^2}{(2.005)^2} = 0.0140\ N$$

Prob. 3



$$T \sin\theta = qE$$

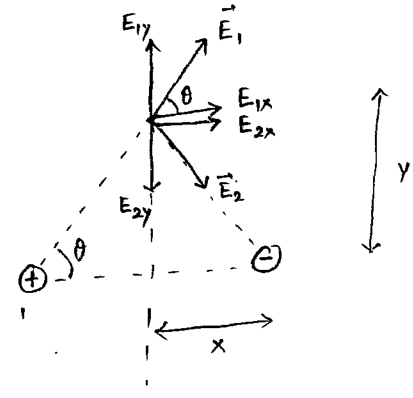
$$T \cos\theta = mg$$

$$\Rightarrow \quad \tan\theta = \frac{qE}{mg}$$

$$q = \frac{mg}{E}\tan\theta$$

$$= \frac{(2.00\times10^{-3}\,\text{kg})(9.8\,\frac{\text{m}}{\text{s}^2})}{(1.00\times10^{3}\,\frac{\text{N}}{\text{C}})}\tan(10.0^\circ) = 3.46\ \mu\text{C}$$

Prob. 4



→ The y-components cancel.

$$\vec{E}_{tot} = 2E_{1x}\,\hat{i} + 0\,\hat{j}$$

$$= 2\frac{kq}{(\sqrt{x^2+y^2})^2}\cos\theta\,\hat{i}$$

$$= 2\frac{kq}{(\sqrt{x^2+y^2})^2}\frac{x}{\sqrt{x^2+y^2}}\,\hat{i}$$

$$= k\frac{2qx}{(x^2+y^2)^{\frac{3}{2}}}\,\hat{i}$$

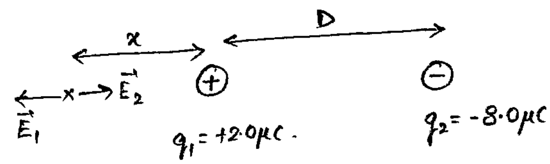
$$= \frac{8.99\times10^{9}\quad 2\times1.0\times10^{-9}\times1.00\times10^{-2}}{\left[(1.00\times10^{-2})^2+(2.50\times10^{-2})^2\right]^{\frac{3}{2}}}\,\hat{i}$$

$$= 9.21\times10^{3}\,\frac{\text{N}}{\text{C}}\,\hat{i}$$

magnitude : $9.21\times10^{3}\,\frac{\text{N}}{\text{C}}$

direction : along $+\hat{x}$.

Prob. 5



→ Argue that the point is on the left of charge q_1.

$$|\vec{E}_1| = |\vec{E}_2|$$

$$\frac{\cancel{k}|q_1|}{x^2} = \frac{\cancel{k}|q_2|}{(D+x)^2}$$

$$x = \frac{D}{\left(\pm\sqrt{\frac{|q_2|}{|q_1|}} - 1\right)} \quad \Rightarrow \quad x = \frac{D}{(\pm 2 - 1)} = +10\,\text{cm} \quad \text{or} \quad \cancel{\boxed{-3.33\,\text{cm}}}$$

Answer: 10 cm to the left of q_1.

Prob. 6

(a) Using $y = \frac{1}{2} g t^2$

$$t_e = \sqrt{\frac{2y}{g}}$$

$$t_p = \sqrt{\frac{2y}{g}}$$

$$\frac{t_p}{t_e} = 1.$$

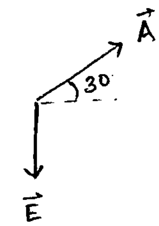
(b) Using $y = \frac{1}{2} a t^2$ $\quad a = \frac{qE}{m}$

$$t_e = \sqrt{\frac{2y}{qE} m_e}$$

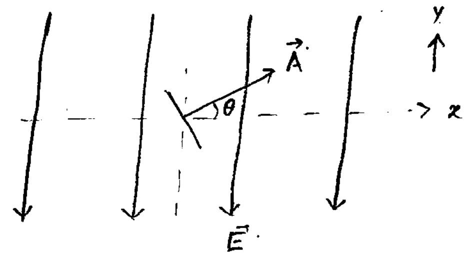
$$t_p = \sqrt{\frac{2y}{qE} m_p}$$

$$\frac{t_p}{t_e} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1836} = 42.9$$

Prob. 7



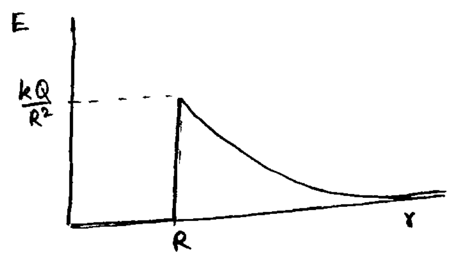
$$A = 0.300 \times 0.400 \ m^2$$
$$= 0.12 \ m^2$$



$$\phi_E = E A \cos(90+30)$$
$$= 4.00 \times 10^3 \ \frac{N}{C} \ 0.12 \ m^2 \ (-0.5)$$
$$= -240 \ \frac{Nm^2}{C}$$

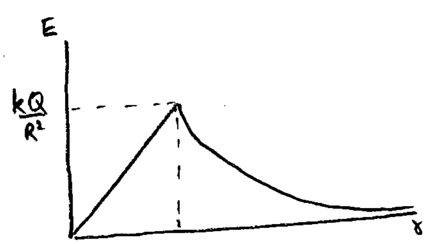
Prob. 8

(a)



$$E = \begin{cases} 0, & r < R, \\ \frac{kQ}{r^2}, & R < r. \end{cases}$$

(b)



$$E = \begin{cases} \frac{kQr}{R^3}, & r < R, \\ \frac{kQ}{r^2}, & R < r. \end{cases}$$