Homework No. 02 (Spring 2017)

PHYS 510: Classical Mechanics

Due date: 2017 Feb 2 (Thursday) 4.30pm

1. (**20 points.**) The eletrostatic energy of a charge distribution $\rho(\mathbf{r})$ is

$$E[\rho] = \frac{1}{2}\int d^3r \int d^3r' \, \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{1}$$

Evaluate

$$\frac{\delta^2 E}{\delta\rho(\mathbf{r})\rho(\mathbf{r}')}. \tag{2}$$

2. (**20 points.**) Consider the action, in terms of the Lagrangian viewpoint,

$$W[\mathbf{x}] = \int_{t_1}^{t_2} dt \left[\frac{1}{2}m\left(\frac{d\mathbf{x}}{dt}\right)^2 - U(\mathbf{x},t)\right]. \tag{3}$$

Assume no variation at the end points t_1 and t_2. Evaluate the functional derivative

$$\frac{\delta W}{\delta \mathbf{x}(t)}. \tag{4}$$

3. (**40 points.**) Consider the following construction in a field theoretical setup

$$W[K] = \frac{1}{2}\int dx \int dx' K(x)\Delta(|x-x'|)K(x'), \tag{5}$$

where W is the action written in terms of a source function $K(x)$ and the Green's function $\Delta(|x-x'|)$. Determine the relation between the corresponding field $\phi(x)$ and the source, by evaluating the functional derivative

$$\phi(x) = \frac{\delta W}{\delta K(x)}. \tag{6}$$

Show that the Green's function satisfies

$$\Delta(|x-x'|) = \frac{\delta^2 W}{\delta K(x)\delta K(x')}. \tag{7}$$

Construct the partition function

$$Z[K] = e^{iW[K]}. \tag{8}$$

Show that

(a) the field satisfies

$$\phi(x) = \frac{1}{i}\frac{\delta \ln Z}{\delta K(x)} \tag{9}$$

(b) and the Green's function is given by

$$\Delta(|x - x'|) = \frac{1}{i}\frac{1}{Z}\frac{\delta^2 Z}{\delta K(x)\delta K(x')}\bigg|_{K=0}. \tag{10}$$