Homework No. 04 (Spring 2017)

PHYS 510: Classical Mechanics

Due date: 2017 Feb 21 (Tuesday) 4.30pm

1. (**40 points.**) Consider the functional

$$W[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x}) \tag{1}$$

constructed out of the function $x = x(t)$ and its derivative $\dot{x} = dx/dt$. In particular, let

$$\frac{\partial L}{\partial t} = 0. \tag{2}$$

(a) Show that

$$\frac{\delta I[x]}{\delta x(t)} = \left[\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}\right] + \Big[\delta(t - t_2) - \delta(t - t_1)\Big]\frac{\partial L}{\partial \dot{x}}. \tag{3}$$

(b) Further, show that

$$\frac{\delta I[x]}{\delta x(t)} = \frac{1}{\dot{x}}\frac{d}{dt}\left(L - \dot{x}\frac{\partial L}{\partial \dot{x}}\right) + \Big[\delta(t - t_2) - \delta(t - t_1)\Big]\frac{\partial L}{\partial \dot{x}}. \tag{4}$$

This property used with the extremum principle is the essence of the Beltrami identity. This also gives us a glimpse of the Legendre transform,

$$H = \dot{x}\frac{\partial L}{\partial \dot{x}} - L. \tag{5}$$

2. (**20 points.**) Write a brief report on the Isoperimetric problem, and the problem of minimal surface of revolution. For example, refer Goldstein, Chapter 2.
A related note: A gyroid is an infinitely connected triply periodic minimal surface discovered by Alan Schoen, who is a retired faculty of the Math department in SIUC and currently a resident of Carbondale.