Homework No. 05 (Spring 2017)

PHYS 510: Classical Mechanics

Due date: 2017 Feb 28 (Tuesday) 4.30pm

1. (**20 points.**) (Refer Goldstein, 2nd edition, Chapter 1 Problem 8.) As a consequence of the Hamilton's stationary action principle, the equations of motion for a system can be expressed as Euler-Lagrange equations,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \tag{1}$$

in terms of a Lagrangian $L(x, \dot{x}, t)$. Show that the Lagrangian for a system is not unique. In particular, show that if $L(x, \dot{x}, t)$ satisfies the Euler-Lagrange equation then

$$L'(x, \dot{x}, t) = L(x, \dot{x}, t) + \frac{dF(x, t)}{dt}, \tag{2}$$

where $F(x, t)$ is any arbitrary differentiable function, also satisfies the Euler-Lagrange equation.

2. (**20 points.**) A mass m_1 is forced to move on a vertical circle of radius R with uniform angular speed ω. Another mass m_2 is connected to mass m_1 using a massless rod of length a, such that it is a simple pendulum with respect to mass m_1. Motion of both the masses are constrained to be in a vertical plane in a uniform gravitational field.

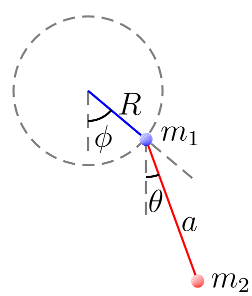

Figure 1: Problem 2.

(a) Write the Lagrangian for the system.

(b) Determine the equation of motion for the system.

(c) Give physical interpretation of each term in the equation of motion.