Solutions

Final Exam (PHYS-205A-002) Fall 2017

Prob. 1

$\Delta y =$? $v_i =$? $a = -9.8 \frac{m}{s^2}$.

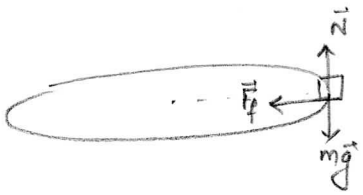
$\Delta t = 3.00\,s$ $v_f = 0$

$$v_f = v_i + a\,\Delta t$$

$$0 = v_i + (-9.8)(3.00) \Rightarrow v_i = 29.4 \frac{m}{s}$$

Prob. 2

(a) $m\vec{g}$ (gravity), $\vec{N}$ (Normal force), and $\vec{F}_f$ (friction).



(b) $|\vec{F}_{tot}| = m\frac{v^2}{R} = (2500)\frac{(14)^2}{250} = 1960$ Newtons. (magnitude)

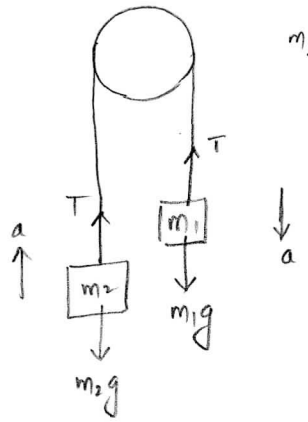
direction: towards the center.

Prob. 3

$m_1 = 32.0\,kg$

$m_2 = 28.0\,kg$

(a)
$$m_1 g - T = m_1 a$$
$$T - m_2 g = m_2 a$$
$$m_1 g - m_2 g = m_1 a + m_2 a$$
$$\Rightarrow a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{32-28}{32+28} 9.8 = 0.65 \frac{m}{s^2}$$



(b) $\Delta y = -3.0\,m$ $v_i = 0$ $a = -9.8 \frac{m}{s^2}$

$\Delta t = ?$ $v_f =$

$$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2 \quad (v_i = 0)$$
$$-3.0 = \frac{1}{2}(-9.8)\Delta t^2$$
$$\Delta t = \sqrt{\frac{(3.0)\,2}{(9.8)}} = 0.78 \text{ second.}$$

Prob. 4

(a) $\frac{dm}{dx} = a$

$$\int dm = \int_0^L a\, dx$$

$$M = aL = 3.00 \frac{kg}{m}(5.00\,m) = 15.0\,kg$$

(b) $$x_{cm} = \frac{\int dm\, x}{\int dm} = \frac{\int_0^L a x\, dx}{\int_0^L a\, dx} = \frac{\frac{L^2}{2}}{L} = \frac{L}{2} = 2.50\,m.$$

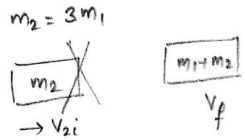
Prob. 5

(a) $W_T = T \Delta x \cos 90 = 0$

(b) $W_g = -\Delta U_g = mgh = (5.0)(9.8)(0.40) = 19.6\,J$

(c) $\Delta K = W_T + W_g = 0 + 19.6\,J = 19.6\,J.$

Prob. 6



$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$m_1 v_{1i} + 3m_1 v_{2i} = 4 m_1 v_f$$

$$v_f = \frac{1}{4}(v_{1i} + 3 v_{2i}) = \frac{1}{4}(4.00 + 3 \times 2.00) = 2.50 \frac{m}{s}$$

Mechanical energy lost $= \Delta K$

$$= \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2$$

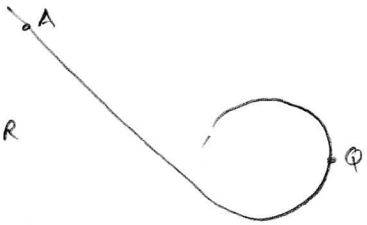
$$= \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} 3m_1 v_{2i}^2 - \frac{1}{2} 4 m_1 v_f^2$$

$$= \frac{1}{2} m_1 \left[v_{1i}^2 + 3 v_{2i}^2 - 4 v_f^2 \right]$$

$$= \frac{1}{2}(2.40 \times 10^{-4}) \left[(4.00)^2 + 3(2.00)^2 - 4(2.50)^2 \right]$$

$$= 3.6 \times 10^{-4}\,J$$

Prob. 7



$\frac{1}{2} m V_A^2 + \frac{1}{2} I \omega_A^2 + mgh_A = \frac{1}{2} m V_Q^2 + \frac{1}{2} I \omega_Q^2 + mg h_Q$

$V_A = 0$, $\omega_A = 0$, $h_A = 7R$, $h_Q = R$

$$mg(7R) = \frac{1}{2} m V_Q^2 + \frac{1}{2} \frac{2}{5} m \underbrace{r^2 \omega_Q^2}_{V_Q^2} + mgR$$

$$6mgR = \frac{1}{2} m r^2 \omega_Q^2 + \frac{1}{5} m r^2 \omega_Q^2$$

$$6gR = \frac{7}{10} r^2 \omega_Q^2$$

$$\omega_Q = \sqrt{\frac{60}{7} \frac{gR}{r^2}}$$

Prob. 8

$I_{2i} = m_2 R^2$

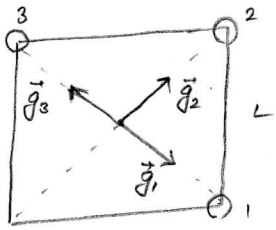
$I_{2f} = 0$

$$I_{1i} \omega_i + I_{2i} \omega_i = I_{1f} \omega_f + \underbrace{I_{2f}}_{=0} \omega_f$$

$$(300)(2.4) + (50)(2.0)^2 (2.4) = 300\, \omega_f$$

$$\omega_f = 2.4 + \frac{50(2.0)^2(2.4)}{300} = 4.0 \ \frac{rad}{s}$$

Prob. 9



$$|\vec{g}_1| = |\vec{g}_2| = |\vec{g}_3| = \frac{GM}{\left(\frac{\sqrt{2}L}{2}\right)^2} = \frac{2GM}{L^2}$$

$\vec{g}_1$ and $\vec{g}_3$ cancel each other.

$\vec{g}_{total} = \vec{g}_1 + \vec{g}_2 + \vec{g}_3 = \vec{g}_2$

magnitude: $|\vec{g}_{total}| = \frac{2GM}{L^2}$

direction: from center towards ②. (45°).

Prob. 10

$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right) = 3.00\cos\left(\frac{2\pi}{4}t + \frac{\pi}{7}\right)$$

(a) $A = 3.00\ \text{cm}$

(b) $f = \frac{1}{T} = \frac{1}{4} = 0.25\ \text{Hz}.$