Homework No. 01 (Fall 2017)

PHYS 440: Quantum Mechanics

Due date: 2017 Aug 31 (Thursday) 4.30pm

1. (**40 points.**) Consider the eigenvalue equation

$$\sigma_x|\sigma'_x\rangle = \sigma'_x|\sigma'_x\rangle, \tag{1}$$

where primes denote eigenvalues.

(a) Find the eigenvalues and normalized eigenvectors (up to a phase) of

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{2}$$

For reference we shall call these eigenvectors $|\sigma'_x = +\rangle$ and $|\sigma'_x = -\rangle$.

(b) Now compute the new matrix

$$\bar{\sigma}_x = \begin{pmatrix} \langle\sigma'_x = +|\sigma_x|\sigma'_x = +\rangle & \langle\sigma'_x = +|\sigma_x|\sigma'_x = -\rangle \\ \langle\sigma'_x = -|\sigma_x|\sigma'_x = +\rangle & \langle\sigma'_x = -|\sigma_x|\sigma'_x = -\rangle \end{pmatrix}. \tag{3}$$

(c) Similarly, compute the new matrices

$$\bar{\sigma}_y = \begin{pmatrix} \langle\sigma'_x = +|\sigma_y|\sigma'_x = +\rangle & \langle\sigma'_x = +|\sigma_y|\sigma'_x = -\rangle \\ \langle\sigma'_x = -|\sigma_y|\sigma'_x = +\rangle & \langle\sigma'_x = -|\sigma_y|\sigma'_x = -\rangle \end{pmatrix}, \quad \text{where} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{4}$$

and

$$\bar{\sigma}_z = \begin{pmatrix} \langle\sigma'_x = +|\sigma_z|\sigma'_x = +\rangle & \langle\sigma'_x = +|\sigma_z|\sigma'_x = -\rangle \\ \langle\sigma'_x = -|\sigma_z|\sigma'_x = +\rangle & \langle\sigma'_x = -|\sigma_z|\sigma'_x = -\rangle \end{pmatrix}, \quad \text{where} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

(d) Find the product of the last two matrices, $\bar{\sigma}_y\bar{\sigma}_z$, and express it in terms of $\bar{\sigma}_x$.

2. (**20 points.**) The Pauli matrix

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

is written in the eigenbasis of σ_z. Write σ_x in the eigenbasis of σ_y.