Homework No. 02 (Fall 2017)

PHYS 440: Quantum Mechanics

Due date: 2017 Sep 7 (Thursday) 4.30pm

1. (**20 points.**) Consider an atom entering a Stern-Gerlach apparatus. Deflection upward begins as soon as the atom enters the inhomogeneous field. By the time the atom leaves the field, it has been deflected upward by a net amount Δz. Compute Δz for

$$\mu_z = 10^{-27}\frac{\text{J}}{\text{G}}, \quad \frac{\partial H_z}{\partial z} = 10^6\frac{\text{G}}{\text{m}}, \quad l = 10\,\text{cm}, \quad \frac{1}{2}mv_x^2 = kT = 10^3\,\text{J}. \tag{1}$$

2. (**20 points.**) Show that

$$p\big([+;0,0] \to [+;\pi,0]\big) = 0. \tag{2}$$

Further, show that

$$p\big([+;0,0] \to [\pm;\theta,\phi] \to [+;\pi,0]\big) = 0, \tag{3}$$

which is a statement of destructive interference. Compare this with the probability for

$$p\big([+;0,0] \to [+;\theta,\phi] \to [+;\pi,0]\big) \tag{4}$$

and

$$p\big([+;0,0] \to [-;\theta,\phi] \to [+;\pi,0]\big). \tag{5}$$

3. (**20 points.**) Using the notation for the probability for a measurement in the Stern-Gerlach experiment, introduced in the class, show that

$$p\big([+;\theta_1,\phi_1] \to [-;\theta_2,\phi_2]\big) = \frac{1-\cos\Theta}{2}, \tag{6}$$

where

$$\cos\Theta = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2). \tag{7}$$

4. (**20 points.**) Using the properties of Pauli matrices,

$$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k, \tag{8}$$

and the Euler formula

$$e^{ix} = \cos x + i\sin x, \tag{9}$$

evaluate

$$e^{-i\theta\frac{\sigma_x}{2}}\sigma_y e^{i\theta\frac{\sigma_x}{2}}. \tag{10}$$

Hint: Use series expansion for the trignometric functions.

5. (**20 points.**) Evaluate

$$[(\boldsymbol{\sigma} \cdot \mathbf{a})(\boldsymbol{\sigma} \cdot \mathbf{b})]\,(\boldsymbol{\sigma} \cdot \mathbf{c}). \tag{11}$$

Then evaluate

$$(\boldsymbol{\sigma} \cdot \mathbf{a})\,[(\boldsymbol{\sigma} \cdot \mathbf{b})(\boldsymbol{\sigma} \cdot \mathbf{c})]\,. \tag{12}$$

Are they equal?