Homework No. 10 (Fall 2017)

PHYS 440: Quantum Mechanics

Due date: 2017 Nov 16 (Thursday) 4.30pm

1. (**20 points.**) We constructed the total angular momentum states of two spin-$\frac{1}{2}$ systems, $j_1 = \frac{1}{2}$, $j_2 = \frac{1}{2}$, by beginning with the total angular momentum state

$$|1,1\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle_{①} \left|\frac{1}{2},\frac{1}{2}\right\rangle_{②} \tag{1}$$

and using the lowering operator to construct the $|1,0\rangle$ and $|1,-1\rangle$ states. The state $|0,0\rangle$ was then constructed (to within a phase factor) as the state orthogonal to $|1,0\rangle$.

   (a) Repeat this exercise by beginning with the total angular momentum state $|1,-1\rangle$ and using the raising operator to construct $|1,0\rangle$ and $|1,1\rangle$ states.

   (b) Investigate the property of the total angular momentum states under the interchange ①↔②. In particular, find out if each of the total angular momentum states are symmetrical (do not change sign) or antisymmetrical (change sign).

2. (**20 points.**) Construct the total angular momentum state $|68,-67\rangle$ for the composite system built out of two angular momenta $j_1 = 31, j_2 = 37$.