Solutions

Midterm Exam 3 (PHYS 205A) Spring 2018

Prob. 1

$$W = \int_i^f \vec{F}\cdot d\vec{x} = \text{Area under } F\text{-versus-}x \text{ curve.}$$

$$= \frac{1}{2}(6-0)(20-10) - \frac{1}{2}(2-0)(30-20)$$

$$= 30 - 10 = 20 \text{ Joules.}$$

Prob. 2

(a) Gravity

$d = 5.0\,m$

(b) $W_g = m\vec{g}\cdot\vec{d}$ $\quad \downarrow mg \quad \downarrow \vec{d}$

$$= mgd\cos(0)$$

$$= (0.60)(9.8)(5.0) = 29 \text{ Joules.}$$

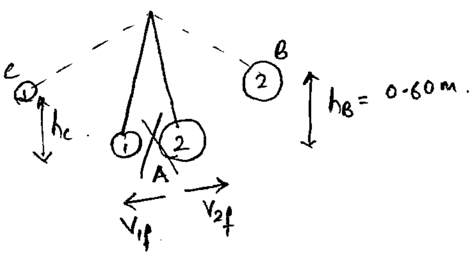
(c) $\Delta U = -W_g = -29\,J$

(d) $\Delta K + \Delta U = 0 \Rightarrow \Delta K = -\Delta U = 29\,J.$

$m_2 =$ Adolf

$m_1 =$ Ed

Prob. 3



energy:

$$\frac{1}{2}m_2 v_{2f}^2 = m_2 g h_B$$

$$v_{2f} = \sqrt{2gh_B} = \sqrt{2(9.8)(0.60)} = 3.4\,\frac{m}{s}$$

momentum:

$$m_1 \underbrace{v_{1i}}_{=0} + m_2 \underbrace{v_{2i}}_{=0} = m_1 v_{1f} + m_2 v_{2f}$$

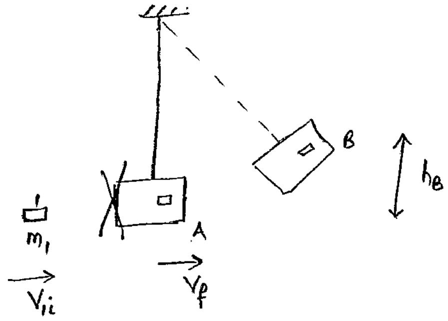
$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{99}{66}\,3.4 = -5.1\,\frac{m}{s}$$

energy

$$\frac{1}{2}\cancel{m_1} v_{1f}^2 = \cancel{m_1} g h_c$$

$$h_c = \frac{v_{1f}^2}{2g} = \frac{(5.1)^2}{2(9.8)} = 1.3\,m$$

Prob. 4



energy:

$$\frac{1}{2}\cancel{(m_1+m_2)}\, V_f^2 = \cancel{(m_1+m_2)}\, g\, h_B$$

$$V_f = \sqrt{2 g h_B} = \sqrt{2(9.8)(0.30)}$$

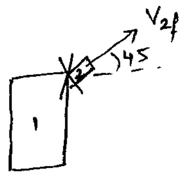
$$= 2.43 \ \frac{m}{s}$$

momentum:

$$m_1 V_{1i} + \underbrace{m_2 V_{2i}}_{=0} = (m_1+m_2)\, V_f$$

$$V_{1i} = \frac{m_1+m_2}{m_1} V_f = \frac{0.0030 + 1.00}{0.0030}\ 2.43 = 810 \ \frac{m}{s}$$

Prob. 5



momentum in x-direct

$$\underbrace{m_1 V_{1i}}_{=0} + \underbrace{m_2 V_{2i}}_{=0} = m_1 V_{1f} + m_2 V_{2f} \cos 45$$

$$V_{1f} = -\frac{m_2}{m_1} V_{2f} \cos 45$$

$$= -\frac{0.00500}{75}(600) \cos 45 = 0.028 \ \frac{m}{s} = 2.8 \ \frac{cm}{s}$$

Prob. 6

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{0 + m_M (60 R_E)}{81 m_M + m_M}$$

$$= \frac{60}{82} R_E = 0.73\ R_E$$

$$= (0.73)\ 6.37 \times 10^6$$

$$= 4.7 \times 10^6 \ m$$

(inside the Earth)

Prob. 7

(a) $$U(r) = \frac{\beta}{2r^2} - \frac{\alpha}{r}$$

$$\frac{\partial U}{\partial r} = -\frac{\beta}{r^3} + \frac{\alpha}{r^2}$$

$$F = -\frac{\partial U}{\partial r} = \frac{\beta}{r^3} - \frac{\alpha}{r^2}$$

$$F = 0 \Rightarrow \frac{\beta}{r_0^3} - \frac{\alpha}{r_0^2} \Rightarrow r_0 = \frac{\beta}{\alpha}$$

(b) $$U_0 = U(r_0)$$

$$= \frac{\beta}{2r_0^2} - \frac{\alpha}{r_0}$$

$$= \frac{\beta}{2}\left(\frac{\alpha}{\beta}\right)^2 - \alpha\left(\frac{\alpha}{\beta}\right) = -\frac{1}{2}\frac{\alpha^2}{\beta}.$$