Midterm Exam No. 02 (Spring 2018)

PHYS 510: Classical Mechanics

Date: 2018 Apr 17

1. (**20 points.**) The Poincaré formula for the addition of (parallel) velocities is, $c = 1$,

$$v = \frac{v_a + v_b}{1 + v_a v_b}, \tag{1}$$

where v_a and v_b are velocities and c is speed of light in vacuum. Assuming that the Poincaré formula holds for all speeds, subluminal ($-1 < v_i < 1$), superluminal ($|v_i| > 1$), and speed of light, analyse what is obtained if you add a subluminal speed to a superluminal speed? That is, is the 'sum' subluminal or superluminal. Is the answer unique?

2. (**20 points.**) Prove that, if p_μ is a time-like vector and $p^\mu s_\mu = 0$, then s^μ is necessarily space-like.

3. (**20 points.**) The Hamiltonian for a Kepler problem is given by

$$H = \frac{p^2}{2\mu} - \frac{\alpha}{r}. \tag{2}$$

Verify that the Hamiltonian H, the angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, and the Laplace-Runge-Lenz vector

$$\mathbf{A} = \frac{\mathbf{r}}{r} - \frac{\mathbf{p} \times \mathbf{L}}{\mu\alpha}, \tag{3}$$

are constants of motion for the Kepler problem. That is, show that

$$\frac{dH}{dt} = 0, \quad \frac{d\mathbf{L}}{dt} = 0, \quad \frac{d\mathbf{A}}{dt} = 0. \tag{4}$$