Homework No. 02 (Spring 2018)

PHYS 510: Classical Mechanics

Due date: Tuesday, 2018 Feb 6, 4.30pm

1. (**20 points.**) (Refer Goldstein, 2nd edition, Chapter 2 Problem 2.) Show that the geodesics on a spherical surface are great circles, that is, circles whose centers lie at the center of the sphere.

   (a) The distance between two points on the surface of a sphere of radius a is characterized by the infinitesimal statement

   $$ds^2 = a^2 d\theta^2 + a^2 \sin^2\theta d\phi^2. \tag{1}$$

   (b) The geodesic is the extremal of the functional

   $$l[y] = \int_{(\theta_1,\phi_1)}^{(\theta_2,\phi_2)} ds = \int_{(\theta_1,\phi_1)}^{(\theta_2,\phi_2)} d\theta \sqrt{1 + \left(\frac{d\phi}{d\theta}\right)^2}. \tag{2}$$

   (c) Since the curve passes through the points (θ_1, ϕ_1) and (θ_2, ϕ_2) we have no variations on the end points. Thus, show that

   $$\frac{1}{r}\frac{\delta l[y]}{\delta\phi(\theta)} = -\frac{d}{d\theta}\left[\frac{\sin^2\theta \dfrac{d\phi}{d\theta}}{\sqrt{1 + \sin^2\theta \left(\dfrac{d\phi}{d\theta}\right)^2}}\right]. \tag{3}$$

   (d) Using the extremum principle show that the differential equation for the geodesic is

   $$\frac{d\phi}{d\theta} = \frac{c}{\sin\theta\sqrt{\sin^2\theta - c^2}}, \tag{4}$$

   where c is an arbitrary constant.

   (e) Solve the differential equation to obtain the equation of geodesic as

   $$\sin(\phi_0 - \phi) = \bar{c}\cot\theta, \tag{5}$$

   where $\bar{c} = c/\sqrt{1-c^2}$ and ϕ_0 is a constant of integration.

   (f) Rewrite the equation of the geodesic in the form

   $$-\sin\phi_0 \sin\theta\cos\phi + \cos\phi_0 \sin\theta\sin\phi + \bar{c}\cos\theta = 0. \tag{6}$$

   Interpret this to be an equation of plane passing through the origin. The condition that this plane has to pass through the two given points determines the constants $\bar{c}$ and ϕ_0, which we shall not attempt here.