Homework No. 04 (Spring 2018)

PHYS 510: Classical Mechanics

Due date: Tuesday, 2018 Feb 20, 4.30pm

1. (**20 points.**) (Refer Goldstein, 2nd edition, Chapter 1 Problem 8.) As a consequence of the Hamilton's stationary action principle, the equations of motion for a system can be expressed as Euler-Lagrange equations,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \tag{1}$$

in terms of a Lagrangian $L(x, \dot{x}, t)$. Show that the Lagrangian for a system is not unique. In particular, show that if $L(x, \dot{x}, t)$ satisfies the Euler-Lagrange equation then

$$L'(x, \dot{x}, t) = L(x, \dot{x}, t) + \frac{dF(x, t)}{dt}, \tag{2}$$

where $F(x, t)$ is any arbitrary differentiable function, also satisfies the Euler-Lagrange equation.

2. (**20 points.**) Refer Figure 1. A mass m_1 is forced to move on a vertical circle of radius R with uniform angular speed ω. Another mass m_2 is connected to mass m_1 using a massless rod of length a, such that it is a simple pendulum with respect to mass m_1. Motion of both the masses is constrained to be in a vertical plane in a uniform gravitational field.

   (a) Write the Lagrangian for the system.

   (b) Determine the equation of motion for the system.

   (c) Give physical interpretation of each term in the equation of motion.

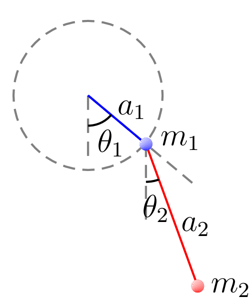

Figure 1: Problem 2.

3. (**20 points.**) A relativistic charged particle of charge q and mass m in the presence of a known electric and magnetic field is described by

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = q\mathbf{E} + \frac{q}{c}\mathbf{v}\times\mathbf{B}. \tag{3}$$

Find the Lagrangian for this system, that implies the equation of motion of Eq. (3), to be

$$L(\mathbf{x},\mathbf{v},t) = -mc^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + \frac{q}{c}\mathbf{v}\cdot\mathbf{A}, \tag{4}$$

using Hamilton's principle of stationary action.