Homework No. 05 (Spring 2018)

PHYS 510: Classical Mechanics

Due date: Tuesday, 2018 Mar 20, 4.30pm

1. (**30 points.**) The motion of a particle of mass m near the Earth's surface is described by

$$\frac{d}{dt}(mv) = -mg, \tag{1}$$

where $v = dz/dt$ is the velocity in the upward z direction.

   (a) Find the Lagrangian for this system that implies the equation of motion of Eq. (1) using Hamilton's principle of stationary action.

   (b) Determine the canonical momentum for this system

   (c) Determine the Hamilton $H(p, z)$ for this system.

   (d) Determine the Hamilton equations of motion.

2. (**30 points.**) The motion of a particle of mass m undergoing simple harmonic motion is described by

$$\frac{d}{dt}(mv) = -kx, \tag{2}$$

where $v = dx/dt$ is the velocity in the x direction.

   (a) Find the Lagrangian for this system that implies the equation of motion of Eq. (2) using Hamilton's principle of stationary action.

   (b) Determine the canonical momentum for this system

   (c) Determine the Hamilton $H(p, x)$ for this system.

   (d) Determine the Hamilton equations of motion.

3. (**30 points.**) Hamiltonian for a charge particle of mass m and charge q in a magnetic field $\mathbf{B}$ is given by

$$H(\mathbf{x}, \mathbf{p}) = \frac{1}{2m}\left(\mathbf{p} - \frac{q}{c}\mathbf{A}\right)^2, \tag{3}$$

where

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{4}$$

Let

$$\frac{\partial \mathbf{A}}{\partial t} = 0. \tag{5}$$

Show that the Hamilton equations of motion leads to the equations, using ($\mathbf{v} = d\mathbf{x}/dt$)

$$m\mathbf{v} = \mathbf{p} - \frac{q}{c}\mathbf{A}, \tag{6a}$$

$$m\frac{d\mathbf{v}}{dt} = \frac{q}{c}\mathbf{v} \times \mathbf{B}. \tag{6b}$$