Homework No. 07 (Spring 2018)

PHYS 510: Classical Mechanics

Due date: Thursday, 2018 Apr 12, 4.30pm

1. (**20 points.**) Lorentz transformation (in one dimension) is given by

$$\Delta z' = \gamma(\Delta z - v\Delta t), \tag{1a}$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c}\frac{\Delta z}{c}\right), \tag{1b}$$

where $\gamma = \sqrt{1 - v^2/c^2}$. Show that for

$$v \ll c \qquad \text{and} \qquad \frac{\Delta z}{\Delta t} \ll c \tag{2}$$

one obtains the Galilean transformation

$$\Delta z' = \Delta z - v\Delta t, \tag{3a}$$

$$\Delta t' = \Delta t. \tag{3b}$$

Note: For the case when Δz and Δt represent the change in position and time of a particle we could have v and $\Delta z/\Delta t$ to be identical.

2. (**60 points.**) The Poincaré formula for the addition of (parallel) velocities is

$$v = \frac{v_a + v_b}{1 + \dfrac{v_a v_b}{c^2}}, \tag{4}$$

where v_a and v_b are velocities and c is speed of light in vacuum. Jerzy Kocik, from the department of Mathematics in SIUC, has invented a geometric diagram that allows one to visualize the Poincaré formula. (Refer [1].) An interactive applet for exploring velocity addition is available at Kocik's web page [2]. (For the following assume that the Poincaré formula holds for all speeds, subluminal ($v_i < c$), superluminal ($v_i > c$), and speed of light.)

   (a) Analyse what is obtained if you add two subluminal speeds?

   (b) Analyse what is obtained if you add a subluminal speed to speed of light?

   (c) Analyse what is obtained if you add a subluminal speed to a superluminal speed?

   (d) Analyse what is obtained if you add speed of light to another speed of light?

   (e) Analyse what is obtained if you add a superluminal speed to speed of light?

   (f) Analyse what is obtained if you add two superluminal speeds?

References

[1] J. Kocik. Geometric diagram for relativistic addition of velocities. *Am. J. Phys.*, 80:737–739, August 2012.

[2] J. Kocik. An interactive applet for exploring relativistic velocity addition. http://lagrange.math.siu.edu/Kocik/relativity/diagram.html.