Midterm Exam No. 01 (Fall 2018)

PHYS 320: Electricity and Magnetism I

Date: 2018 Sep 14

1. (**20 points.**) Using the property of Kronecker δ-function and Levi-Civita symbol in three dimensions evaluate the following using index notation.

$$\delta_{ij}\delta_{ji} = \tag{1a}$$

$$\delta_{ij}\varepsilon_{ijk} = \tag{1b}$$

$$\varepsilon_{ijm}\delta_{mn}\varepsilon_{nij} = \tag{1c}$$

2. (**20 points.**) Using index notation and the properties of Kronecker δ-function and Levi-Civita symbol in three dimensions expand the left hand side of the vector equation below to express it in the form on the right hand side,

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = \alpha(\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) + \beta(\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C}). \tag{2}$$

In particular find the numbers α and β.

3. (**20 points.**) Evaluate the left hand side of the equation

$$\boldsymbol{\nabla}\frac{1}{r^3} = \alpha\,\hat{\mathbf{r}}\,r^n. \tag{3}$$

Thus find α and n.

4. (**20 points.**) Evaluate the integral

$$\int_{-1}^{1} dx\,\delta(1-2x)\Big[8x^2+2x-1\Big]. \tag{4}$$

(Caution: Be careful to avoid a possible error in sign.)

5. (**20 points.**) A uniformly charged infinitely thin circular disc of radius R and total charge Q is placed on the x-y plane such that the normal vector is along the z axis and the center of the disc at the origin. Write down the charge density of the disc in terms of δ-function(s) and Heaviside step function(s).

6. (**20 points.**) Evaluate the flux,

$$\int_S d\mathbf{a}\cdot\mathbf{E}, \tag{5}$$

of the uniform (homogeneous in space) field

$$\mathbf{E} = E\,\hat{\mathbf{z}} \tag{6}$$

through the surface of a circular disc of radius R placed on the x-y plane.