(Preview of) Midterm Exam No. 02 (Fall 2018)

PHYS 320: Electricity and Magnetism I

Date: 2018 Oct 12

1. (**20 points.**) Consider an electric line-dipole at the origin, constituting of an infinitely long and infinitely thin rod with uniform positive line charge density λ (charge/length), parallel to the z axis, at $x = a$, and another such rod with negative line charge density at $x = -a$. Together these form an electric line-dipole moment $\boldsymbol{\beta} = 2a\lambda\,\hat{\mathbf{i}}$. The electric potential due to this line-dipole at the point

$$\boldsymbol{\rho} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}, \qquad \rho = \sqrt{x^2 + y^2}, \tag{1}$$

is given by the expression

$$\phi(\boldsymbol{\rho}) = -\frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{(x-a)^2 + y^2}{(x+a)^2 + y^2}\right]. \tag{2}$$

   (a) For $a \ll \rho$ show that the potential is approximately given by

$$\phi(\boldsymbol{\rho}) = \frac{1}{4\pi\varepsilon_0} \frac{2\beta x}{(x^2 + y^2)}. \tag{3}$$

   (b) Consider the limit when a is made to vanish while λ becomes infinite, in such a way that $2a\lambda$ remains the finite value β. This is a point line-dipole. The electric potential for a point line-dipole is exactly described by Eq. (3). Using cylindrical polar coordinates write $z = \rho\cos\phi$ and thus rewrite the potential of a point dipole in Eq. (3) in the form

$$\phi(\boldsymbol{\rho}) = \frac{1}{2\pi\varepsilon_0} \frac{\beta\cos\phi}{\rho^2} = \frac{1}{2\pi\varepsilon_0} \frac{\boldsymbol{\beta}\cdot\boldsymbol{\rho}}{\rho^2}. \tag{4}$$

   (c) Evaluate the electric field due to a point line-dipole using

$$\mathbf{E} = -\boldsymbol{\nabla}\phi. \tag{5}$$

   Draw the electric field lines of a point line-dipole for $\boldsymbol{\beta} = \beta\,\hat{\mathbf{i}}$. Then, draw the equipotential lines. Are the equipotential lines circular?

2. (**20 points.**) In a homework problem (Problem 2, HW-05) we learned that the charge density

$$\rho(\mathbf{r}) = \frac{\sigma}{r}, \qquad r = \sqrt{x^2 + y^2 + z^2}, \tag{6}$$

creates a uniform, spherically symmetric, pointing radially ourward from the origin, electric field

$$\mathbf{E}(\mathbf{r}) = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}, \qquad \mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}. \tag{7}$$

(a) Verify this by computing $\boldsymbol{\nabla} \cdot \varepsilon_0 \mathbf{E}$ for the electric field in Eq. (7). Draw these electric field lines, keeping in mind that the density of electric field lines relates to the intensity of electric field lines.

(b) Next, determine what charge density will create a uniform, cylindrically symmetric, pointing radially ourward from the symmetry axis of cylinder, electric field

$$\mathbf{E}(\mathbf{r}) = \frac{\sigma}{\varepsilon_0}\hat{\boldsymbol{\rho}}, \qquad \boldsymbol{\rho} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}. \tag{8}$$

Draw these electric field lines.

Caution: The Greek letter ρ is used to represent the charge density and the cylindrical coordinate.

3. (**20 points.**) Not available in preview mode.

4. (**20 points.**) Not available in preview mode.

5. (**20 points.**) Not available in preview mode.