Midterm Exam No. 02 (Fall 2018)

PHYS 320: Electricity and Magnetism I

Date: 2018 Oct 15

1. (**20 points.**) Consider an electric line-dipole at the origin, constituting of an infinitely long and infinitely thin rod with uniform positive line charge density λ (charge/length), parallel to the z axis, at $x = a$, and another such rod with negative line charge density at $x = -a$. Together these form an electric line-dipole moment $\boldsymbol{\beta} = 2a\lambda\,\hat{\mathbf{i}}$. The electric potential due to this line-dipole at the point

$$\boldsymbol{\rho} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}, \qquad \rho = \sqrt{x^2 + y^2}, \tag{1}$$

is given by the expression

$$\phi(\boldsymbol{\rho}) = -\frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{(x-a)^2 + y^2}{(x+a)^2 + y^2}\right]. \tag{2}$$

(a) For $a \ll \rho$ show that the potential is approximately given by

$$\phi(\boldsymbol{\rho}) = \frac{1}{4\pi\varepsilon_0} \frac{2\beta x}{(x^2 + y^2)}. \tag{3}$$

(b) Consider the limit when a is made to vanish while λ becomes infinite, in such a way that $2a\lambda$ remains the finite value β. This is a point line-dipole. The electric potential for a point line-dipole is exactly described by Eq. (3). Using cylindrical polar coordinates write $x = \rho\cos\phi$ and thus rewrite the potential of a point dipole in Eq. (3) in the form

$$\phi(\boldsymbol{\rho}) = \frac{1}{2\pi\varepsilon_0} \frac{\beta x \cos\phi}{\rho^2} = \frac{1}{2\pi\varepsilon_0} \frac{\boldsymbol{\beta} \cdot \boldsymbol{\rho}}{\rho^2}. \tag{4}$$

(c) Evaluate the electric field due to a point line-dipole using

$$\mathbf{E} = -\boldsymbol{\nabla}\phi. \tag{5}$$

Draw the electric field lines of a point line-dipole for $\boldsymbol{\beta} = \beta\,\hat{\mathbf{i}}$. Then, draw the equipotential lines. Are the equipotential lines circular?

2. (**20 points.**) In a homework problem (Problem 2, HW-05) we learned that the charge density

$$\rho(\mathbf{r}) = \frac{\sigma}{r}, \qquad r = \sqrt{x^2 + y^2 + z^2}, \tag{6}$$

creates a uniform, spherically symmetric, pointing radially ourward from the origin, electric field

$$\mathbf{E}(\mathbf{r}) = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}, \qquad \mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}. \tag{7}$$

(a) Verify this by computing $\boldsymbol{\nabla}\cdot\varepsilon_0\mathbf{E}$ for the electric field in Eq. (7). Draw these electric field lines, keeping in mind that the density of electric field lines relates to the intensity of electric field.

(b) Next, determine what charge density will create a uniform, cylindrically symmetric, pointing radially ourward from the symmetry axis of cylinder, electric field

$$\mathbf{E}(\mathbf{r}) = \frac{\sigma}{\varepsilon_0}\hat{\boldsymbol{\rho}}, \qquad \boldsymbol{\rho} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}. \tag{8}$$

Draw these electric field lines.

Caution: The Greek letter ρ is used to represent the charge density and the cylindrical coordinate.

3. (**20 points.**) Two electrons and two protons are placed at the corners of a square of side a, such that the electrons are at diagonally opposite corners.

   (a) What is the electric potential at the center of square?

   (b) What is the electric potential at the midpoint of either one of the sides of the square?

   (c) How much potential energy is required to move another proton from infinity to the center of the square?

   (d) How much additional potential energy is required to move the proton from the center of the square to one of the midpoint of either one of the sides of the square?

4. (**20 points.**) (Griffiths 4th edition, Problem 2.32) Two positive charges, q_1 and q_2 (masses m_1 and m_2) are at rest, held together by a massless string of length a. Now the string is cut, and the particles fly off in opposite directions. How fast is each one going, when they are far apart?

5. (**20 points.**) Consider two concentric spherical (perfectly) conducting shells, of radii a and $b > a$. The inner shell has a charge $+Q$ and the outer shell has a charge $-Q$.

   (a) Determine the expression for electric field everywhere.

   (b) Plot the magnitude of electric field as a function of the distance from the center of the concentric shells.

   (c) What is force experienced by another charge $+q$ a distance r from the center?

   (d) Plot the electric potential as a function of distance, choosing the the potential at the center to be zero.