Midterm Exam No. 03 (Fall 2018)

PHYS 320: Electricity and Magnetism I

Date: 2018 Nov 12

1. (**20 points.**) Fourier series (or transformation) is defined as ($0 \leq \theta < 2\pi$)

$$f(\theta) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{im\theta} a_m, \tag{1}$$

where the coefficients a_m are determined using

$$a_m = \int_0^{2\pi} d\theta \, e^{-im\theta} f(\theta). \tag{2}$$

Determine all the Fourier coefficients a_m for

$$f(\theta) = \cos\theta = \frac{(e^{i\theta} + e^{-i\theta})}{2}. \tag{3}$$

2. (**20 points.**) Consider the inhomogeneous linear differential equation

$$\left(a\frac{d^2}{dx^2} + b\frac{d}{dx} + c \right) f(x) = \delta(x). \tag{4}$$

Use the Fourier transformation and the associated inverse Fourier transformation

$$f(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx} \tilde{f}(k), \tag{5a}$$

$$\tilde{f}(k) = \int_{-\infty}^{\infty} dx e^{-ikx} f(x), \tag{5b}$$

to show that the corresponding equation satisfied by $\tilde{f}(k)$ is algebraic. Find $\tilde{f}(k)$.

3. (**20 points.**) Consider two grounded, thin, perfect conductors occupying half planes extending radially outward from the z axis. Let these planes intersect at the z axis making an angle of 120° between them. That is, say, the two planes are $\theta = \pi/3$ and $\theta = -\pi/3$. Place a point charge on the plane $\theta = \pi/6$ as described in Figure 1, to the left. The resulting image charge configuration, assuming that the method of images extends to these configurations analogous to optical images in a mirror is shown in Figure 1, to the right.

   Let us vary the position of the point charge slightly such that it is on the plane $\theta = (\pi/6) + \varepsilon$, where $\varepsilon > 0$. Find the resulting variation in the image charge configuration.

4. (**20 points.**) A grounded perfectly conducting plate is placed at $z = 0$ plane. A negative charge of magnitude q is placed at $\mathbf{r} = d\,\hat{\mathbf{z}}$. Using method of images determine the direction and magnitude of the electric field at the point $\mathbf{r} = 2d\,\hat{\mathbf{z}}$.

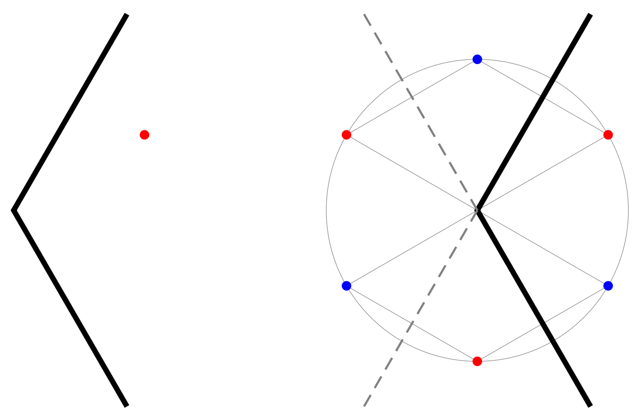

Figure 1: A charge near two intersecting grounded perfect conductors.