Homework No. 01 (2018 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Monday, 2018 Aug 27, 2:00 PM, in class

1. (**10 points.**) (Refer Problem 1.2, Griffiths 4th edition.)
   Is the cross product associative?
   $$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \stackrel{?}{=} (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}. \tag{1}$$
   If so, prove it; If not, provide a counterexample.

2. (**10 points.**) (Based on Example 1.2, Griffiths 4th edition.)
   Draw a cube with its eight vertex corners having coordinates [(0,0,0), (1,0,0), (0,1,0), (1,1,0), (0,0,1), (1,0,1), (0,1,1), (1,1,1)] such that its edges overlap each of the axes. Find the angle between the face diagonal obtained by connecting $(1,0,1) \to (1,1,1)$, and the body diagonal obtained by connecting $(0,0,0) \to (1,1,1)$, of the cube.

3. (**10 points.**) (Based on Example 1.4, Griffiths 4th edition.)
   Draw a tetrahedron, (pyramid with four triangular faces,) by connecting the vertex corners [(0,0,0), (2,0,0), (0,3,0), (0,0,5)]. Use the cross product to find the components of the unit vector $\hat{\mathbf{n}}$ perpendicular to the triangular face of the tetrahedron obtained by connecting the coordinates [(2,0,0), (0,3,0), (0,0,5)].

4. (**10 points.**) Using index notation and the antisymmetric property of the Levi-Civita symbol show that
   $$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = -\mathbf{A} \cdot \mathbf{C} \times \mathbf{B}. \tag{2}$$

5. (**20 points.**) In three dimensions the Levi-Civita symbol is given in terms of the determinant of the Kronecker delta,
   $$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix} \tag{3a}$$
   $$= \delta_{il}(\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}) - \delta_{im}(\delta_{jl}\delta_{kn} - \delta_{jn}\delta_{kl}) + \delta_{in}(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}). \tag{3b}$$
   Using the above identity show that
   $$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}. \tag{4}$$
   Thus, derive the vector identity (using index notation)
   $$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}). \tag{5}$$