

Homework No. 03 (2018 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2018 Sep 7, 2:00 PM, in class

1. **(10 points.)** (Based on problem 1.26 Griffiths 4th edition.)
Calculate the Laplacian of the function

$$T_a = x^2 + 2xy + 3z + 4. \quad (1)$$

2. **(10 points.)** (Based on problem 1.32/1.31 Griffiths 4th/3rd edition.)
Check the fundamental theorem for gradients,

$$\int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{l} \cdot \nabla T = T(\mathbf{b}) - T(\mathbf{a}), \quad (2)$$

using $T = x^2 + 4xy + 2yz^3$, the points $\mathbf{a} = (0, 0, 0)$, $\mathbf{b} = (1, 1, 1)$, integrated along the path in Fig. 1.28 (a) in Griffiths, obtained by connecting the points

$$(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1). \quad (3)$$

3. **(10 points.)** (Based on problem 1.33/1.32 Griffiths 4th/3rd edition.)
Check the fundamental theorem of divergence,

$$\int_V d^3x \nabla \cdot \mathbf{E} = \oint_S d\mathbf{a} \cdot \mathbf{E}, \quad (4)$$

for the vector field $\mathbf{E} = x \hat{\mathbf{x}}$. Take a cube of length L as your volume, which is placed with one edge parallel to the x -axis. Using the fact that the divergence of a vector field at a point tells us whether a point is a source or sink of the field, estimate the distribution of the source and sink for the field \mathbf{E} ?

4. **(10 points.)** (Based on problem 1.34/1.33 Griffiths 4th/3rd edition.)
Check the fundamental theorem of curl,

$$\int_S d\mathbf{a} \cdot \nabla \times \mathbf{E} = \oint_C d\mathbf{l} \cdot \mathbf{E}, \quad (5)$$

(where the sense of the line integration is given by the right hand rule: the contour C is traversed in the sense of the fingers of the right hand and the thumb points in the sense of the orientation of the surface,) for the vector field $\mathbf{E} = y \hat{\mathbf{x}} + z \hat{\mathbf{y}} + x \hat{\mathbf{z}}$. Take a square of length L on the $z = 0$ plane as your surface, which is placed with one side parallel to the x -axis. Using the fact that the curl of a vector field at a point is a measure of the torque experienced by a (point) dipole at the point, estimate the torque field.