(Optional) Homework No. 06 (2018 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2018 Nov 30, 2:00 PM, in class

1. (**20 points.**) Assume Earth to be a solid spherical ball of uniform density. Consider a hypothetical tunnel passing through the center of Earth and connecting two points on the surface of Earth by a straight line. Determine the time taken, (in minutes) to two siginificant digits, starting from rest, to travel from one point to the other, when a mass is dropped at one end of the tunnel. Ignore friction and the rotational motion of Earth. Use the mass of Earth to be 6.0×10^{24} kg, radius of Earth to be 6.4×10^{6} m. Newton's gravitational constant is $6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$.

2. (**40 points.**) A more realistic density profile of Earth is

$$\rho(r) = \begin{cases} \rho_0, & \text{for} \quad r < \frac{R}{2}, \\ \frac{1}{2}\rho_0, & \text{for} \quad \frac{R}{2} < r < R, \end{cases} \tag{1}$$

where

$$\rho_0 = \frac{16}{9} \frac{M}{\frac{4\pi}{3} R^3}, \tag{2}$$

where R is the radius of Earth and M is the mass of Earth. Show that the above density profile leads to the following profile for the gravitational field for Earth,

$$g(r) = \begin{cases} -\dfrac{16}{9}\dfrac{GMr}{R^3}, & \text{for} \quad r < \dfrac{R}{2}, \\ -\dfrac{8}{9}\dfrac{GM}{R^2}\dfrac{1}{2}\left[\dfrac{2r}{R} + \left(\dfrac{R}{2r}\right)^2\right], & \text{for} \quad \dfrac{R}{2} < r < R, \\ -\dfrac{GM}{r^2}, & \text{for} \quad R < r, \end{cases} \tag{3}$$

where G is Newton's gravitational constant. Plot $g(r)$ as a function of r. Approximate the above gravitational field as

$$g(r) \approx \begin{cases} -\dfrac{GM}{R^2}\dfrac{2r}{R}, & \text{for} \quad r < \dfrac{R}{2}, \\ -\dfrac{GM}{R^2}, & \text{for} \quad \dfrac{R}{2} < r < R, \\ -\dfrac{GM}{r^2}, & \text{for} \quad R < r. \end{cases} \tag{4}$$

Plot the approximate gravitational field and compare it with the exact version. Argue that it is accurate to about ten percent. Determine the new time taken, (in minutes) to two siginificant digits, starting from rest, to travel from one point to the other, when a mass is dropped at one end of the tunnel. Ignore friction and the rotational motion of Earth.
Refer: The gravity tunnel in a non-uniform Earth, by Alexander R. Klotz, Am. J. Phys. 83 (2015) 231; arXiv:1308.1342.