Homework No. 11 (2018 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2018 Nov 16, 2:00 PM, in class

1. (**20 points.**) Using Mathematica (or another graphing tool) plot the Legendre polynomials $P_l(x)$ for $l = 0, 1, 2, 3, 4$ on the same plot. Note that $-1 \leq x \leq 1$. Based on the pattern you see what can you conclude about the number of roots for $P_l(x)$.
   Hint: In Mathematica these plots are generated using the following commands:
   `Plot[{LegendreP[0,x], LegendreP[1,x], LegendreP[2,x], LegendreP[3,x], LegendreP[4,x] },{x,-1,1}]`
   Compare your plots with those in Wikipedia article on 'Legendre Polynomials'. While there read the Wikipedia article on Adrien-Marie Legendre and the associated 'Portrait Debacle'.

2. (**10 points.**) Legendre polynomials $P_l(x)$ satisfy the relation

$$\int_{-1}^{1} dx\, P_l(x) = 0 \quad \text{for} \quad l \geq 1. \tag{1}$$

   Verify this explicitly for $l = 1, 2, 3, 4$.

3. (**20 points.**) The induced charge on the surface of a spherical conducting shell of radius a due to a point charge q placed a distance b away from the center is given by

$$\rho(\mathbf{r}) = \sigma(\theta, \phi)\, \delta(r - a), \tag{2}$$

   where

$$\sigma(\theta, \phi) = -\frac{q}{4\pi a} \frac{(r_>^2 - r_<^2)}{(a^2 + b^2 - 2ab\cos\theta)^{\frac{3}{2}}}, \tag{3}$$

   where $r_< = \text{Min}(a, b)$ and $r_> = \text{Max}(a, b)$. Calculate the dipole moment of this charge configuration (excluding the original charge q) using

$$\mathbf{d} = \int d^3r\, \mathbf{r}\, \rho(\mathbf{r}), \tag{4}$$

   for the two cases $a < b$ and $a > b$, representing the charge being inside or outside the sphere.