# Final Exam (Fall 2018)

## PHYS 500A: Mathematical Methods

Date: 2018 Dec 11

1. (**20 points.**) Evaluate the integral

$$\frac{1}{2\pi i}\oint_c \frac{dz}{(z-a)} \tag{1}$$

for complex valued $a$, where the contour $c$ is along the unit circle going counterclockwise.

2. (**20 points.**) Verify that

$$g(z) = \frac{1}{2k}\, e^{-k|z|} \tag{2}$$

is a particular solution of the differential equation

$$\left(-\frac{d^2}{dz^2} + k^2\right) g(z) = \delta(z). \tag{3}$$

3. (**20 points.**) In a homework we verified the relation

$$(\mathbf{a}_2 \cdot \boldsymbol{\nabla})(\mathbf{a}_1 \cdot \boldsymbol{\nabla})\frac{1}{r} = \frac{1}{r^5}\Big[3(\mathbf{a}_2 \cdot \mathbf{r})(\mathbf{a}_1 \cdot \mathbf{r}) - (\mathbf{a}_2 \cdot \mathbf{a}_1)r^2\Big], \tag{4}$$

which is also a solution to Laplace's equation for $\mathbf{r} \neq 0$, that need not be verified here. Evaluate

$$(\mathbf{a}_3 \cdot \boldsymbol{\nabla})(\mathbf{a}_2 \cdot \boldsymbol{\nabla})(\mathbf{a}_1 \cdot \boldsymbol{\nabla})\frac{1}{r}. \tag{5}$$

4. (**20 points.**) Use the integral representation of $J_m(t)$,

$$i^m J_m(t) = \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{it\cos\alpha - im\alpha}, \tag{6}$$

to prove the recurrence relation

$$2\frac{d}{dt} J_m(t) = J_{m-1}(t) - J_{m+1}(t). \tag{7}$$