Homework No. 06 (Fall 2018)

PHYS 520A: Electromagnetic Theory I

Due date: Monday, 2018 Nov 5, 4.00pm

1. (**30 points.**) Conducting electrons, unlike bound electrons, are not confined to a particular atom. In the Drude model the conduction electrons are described by Newton's law

$$m\frac{d}{dt}\mathbf{v}(t) = -m\gamma\mathbf{v}(t) + e\mathbf{E}(t), \tag{1}$$

where the effects of collisions are modeled by a frictional force proportional (and opposite) to the velocity. If n_f is the (constant) density of (free) conduction electrons, then the conduction current density is given by

$$\mathbf{J}(t) = n_f e\mathbf{v}(t). \tag{2}$$

(a) Solve the above differential equation and express the solution in the form

$$\mathbf{v}(t) = \frac{e}{m}\int_{-\infty}^{t} dt'\, e^{-\gamma(t-t')}\mathbf{E}(t'). \tag{3}$$

(b) For a constant electric field show that the current density is given by

$$\mathbf{J} = \frac{n_f e^2}{m\gamma}\mathbf{E}. \tag{4}$$

(c) Use the statement of Ohm's law,

$$\mathbf{J} = \sigma\mathbf{E}, \tag{5}$$

to identify the expression for static conductivity σ.

(d) Find the current density for

$$\mathbf{E}(t) = \mathbf{E}_0\frac{1}{\gamma_0}\delta(t), \tag{6}$$

if $\mathbf{J}(t) = 0$ for $t < 0$. In particular, what is $\mathbf{J}$ immediately after $t = 0$?

2. (**40 points.**) Plot the following as a function of ω:

(a) Re$\chi(\omega)$ for an insulator ($\gamma \ll \omega_0$) in the Drude-Lorentz dielectric model.

(b) Im$\chi(\omega)$ for an insulator ($\gamma \ll \omega_0$) in the Drude-Lorentz dielectric model.

(c) Re$\chi(\omega)$ for a metal ($\omega_0 \ll \gamma$) in the Drude-Lorentz dielectric model.

(d) Im$\chi(\omega)$ for a metal ($\omega_0 \ll \gamma$) in the Drude-Lorentz dielectric model.

Observe that the real part of the dielectric function (square of refractive index) represents dispersion. Anomalous dispersion is the behavior when the refractive index decreases with increase in frequency. Imaginary part of the dielectric function represents absorption. Is it correct to state that anomalous dispersion is accompanied by absorption?

3. (**20 points.**) Calculate the plasma frequency of gold using

$$\omega_p^2 = \frac{n_f e^2}{m\varepsilon_0}, \tag{7}$$

where n_f is the density of conduction electrons. Is this greater than or less than the frequency spectrum of visible light? Are good conductors always opaque and shiny to human eye?