Midterm Exam No. 01 (Spring 2019)

PHYS 301: Theoretical Methods in Physics

Date: 2019 Feb 8

1. (**20 points.**) Find the real and imaginary part of the function

$$f = \ln z. \tag{1}$$

2. (**20 points.**) Find the sixth roots of unity by solving the equation

$$z^6 = 1. \tag{2}$$

   Mark the the points corresponding to the six roots on the complex plane.

3. (**20 points.**) Hyperbolic cosine and sine are defined in terms of the exponential function,

$$\cosh x = \frac{e^x + e^{-x}}{2}, \tag{3a}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}. \tag{3b}$$

   Using the above prove the identity

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y. \tag{4}$$

4. (**20 points.**) Check if the function

$$f(z) = e^z + e^{iz} \tag{5}$$

   satisfies the Cauchy-Riemann conditions.

5. (**20 points.**) Evalauate the contour integral

$$I = \frac{1}{2\pi i} \oint_c dz \frac{e^{iz}}{\left(z^2 + \frac{1}{4}\right)}, \tag{6}$$

   where the contour c is a unit circle going counterclockwise with center at the origin.